Astronomy 102, Fall 2003

Exam 2 Solutions

Note that I include some commentary on some of these problems. This is *not* to say that I expected you to write this much on the exam. I merely do this to help explain why I graded the way I did, to help you understand why the answer is what it is, or to let you know what many of you did wrong.

1. (c) and (e)

2. (c)

3. (c)

4. (a)

5. (a)

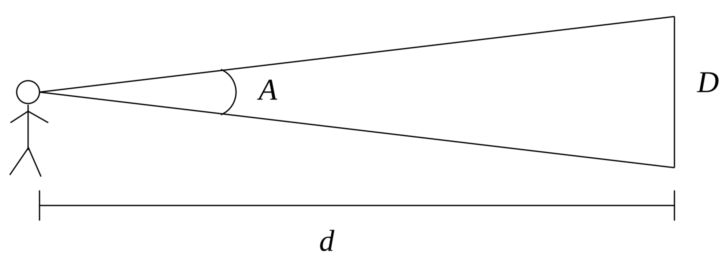
6. (a)

7. (d)

8. (a), (b), (c), and (d)

9. Very few of you got this. The key things to think about where given to you in th eproblem statement. Think about what pictures of stars and galaxies look like; keep in mind what we mean when we say how big something looks.

Take these two statements, and look at the picture of NGC6744 on the front of the test. Note that NGC6744 is extended across the whole picture, but the stars in our galaxy are very small. What *do* we mean when we say how big something looks? Why does the moon "look" bigger than the stars in the sky? It's the angular size. In fact, *that* picture was also on the front of the test. A slightly different version of it:



The small angle formula tells us $A = D/d$. D is the diameter— the size— and d is the distance; A is the observed angular diameter, or how big the object looks. In other words, the angular diameter *is* the size/distance ratio! Simply because galaxies we see have bigger angular diameters than stars, that tells us immediately that galaxies have a larger size/distance ratio— that is, they are typically larger compared to how far away they are than stars are!

10. What we know (using parallax to find distances):

$$d_A = \frac{1}{0.10}\,\text{pc} \qquad d_B = \frac{1}{0.025}\,\text{pc}$$
$$F_B = 2\,F_A$$

We want luminosities; how can we related luminosity to quantities we know?

$$F_A = \frac{L_A}{4\,\pi\,{d_A}^2} \qquad F_B = \frac{L_B}{4\,\pi\,{d_B}^2}$$

Solve these two equations for luminosity by multiplying both sides by $4\pi\,d^2$:

$$L_A = 4\,\pi\,{d_A}^2\,F_A \qquad L_B = 4\,\pi\,{d_B}^2\,F_B$$

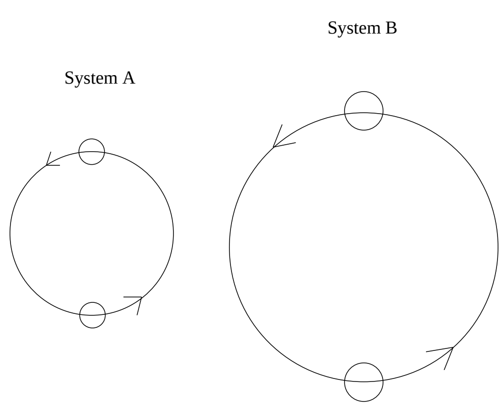
We want the ratio of luminosities, so divide these two; notice that the 4π will cancel out:

$$\frac{L_A}{L_B} = \left(\frac{d_A}{d_B}\right)^2 \left(\frac{F_A}{F_B}\right)$$
$$\frac{L_A}{L_B} = \left(\frac{1/0.10\,\text{pc}}{1/0.025\,\text{pc}}\right)^2 \left(\frac{F_A}{2\,F_A}\right)$$
$$\frac{L_A}{L_B} = \left(\frac{0.025}{0.10}\right)^2 \left(\frac{1}{2}\right)$$
$$\boxed{\frac{L_A}{L_B} = 0.03125 = \frac{1}{32}}$$

(Really, just 0.31 to two significant figures.)

11. What do we know? The systems have the same period; it takes the same amount of time for the two to orbit. Both are in circular orbits. In each system, the two stars have the same mass. Note that when stars are in a circular orbit, they orbit around the center of mass. (In an elliptical orbit, the center of mass is the focus of the orbit, but is not in the center.) Because, in each system, the two stars have the same mass, the center of mass is going to be exactly between the two stars; this will be the point both stars orbit around.

Will the size of the orbits be the same? You might think so, since they have the same period. However, think what would happen if this were so. System B has *more* massive stars; if the size of the orbits were the same, then there would be a *greater* gravitational force between the two stars in that system. To counteract that force and stay in orbit, the stars would then have to be moving faster…which would mean that they would get around the orbit faster. Yet, we know they take the same amount of time as System A to get around their orbits. Thus, the orbits of System B must be bigger.



Note that you didn't need to write all that, only draw the right picture!

12.

(a) Remember that there are *two* photons, but also *two* particles. The energy in the mass of these two particles is converted into the energy of the photons:

$$2\,E \;=\; 2\,m\,c^2$$
$$E \;=\; m\,c^2$$
$$E \;=\; (9.1 \times 10^{-31}\,\text{kg})\,(3.0 \times 10^{8}\,\frac{\text{m}}{\text{s}})^2$$
$$\boxed{E \;=\; 8.2 \times 10^{-14}\,\text{J}}$$

(b)

$$E \;=\; h\,f \qquad f\,\lambda \;=\; c$$
$$f \;=\; \frac{E}{h} \qquad \lambda \;=\; \frac{c}{f}$$

Put these two together:

$$\lambda \;=\; \frac{h\,c}{E}$$
$$\lambda \;=\; \frac{(6.626 \times 10^{-34}\,\text{J s})\,(3.0 \times 10^{8}\,\text{m/s})}{8.19 \times 10^{-14}\,\text{J}}$$
$$\boxed{\lambda \;=\; 2.4 \times 10^{-12}\,\text{m}}$$

(c) Start by figuring out the energy of one blue photon, using the equation from the previous part:

$$E_{\text{blue}} \;=\; \frac{h\,c}{4500\,\text{Å}}$$

Make sure to get these into consistent units! 4500 Å$= 4500 \times 10^{-10}$ m.

$$E_{\text{blue}} \;=\; \frac{(6.626 \times 10^{-34}\,\text{J s})\,(3.0 \times 10^{8}\,\text{m/s})}{4500 \times 10^{-10}\,\text{m}}$$
$$E_{\text{blue}} \;=\; 4.42 \times 10^{-17}\,\text{J}$$

To figure out how many blue photons have the same energy as one of

(d) Note that the fact that it is blue light doesn't actually matter; all you want is luminosity, which is energy produced per second. So there's no need to worry about the frequency of blue light or any such.

$$L \;=\; \frac{E}{t} \;=\; \frac{2\,m\,c^2}{1\,\text{s}}$$

The 2 is because we have 1ng of electrons and 1ng of positrons, for 2ng total (where m is 1ng).

$$L \;=\; \frac{2\,(10^{-12}\,\text{kg})\,(3 \times 10^{8}\,\text{m/s})^2}{1\,\text{s}}$$
$$L \;=\; 180,000\,\text{W}$$

This is way the heck more than a lightbulb. Using tiny amounts of matter/antimatter gets you a lot of power. This shows two things; first, that mass is a very efficient way to store energy. Second, that all those science fiction writers who think that starships might use antimatter as fuel might be onto something.

13. **(a)** Because this star is just like the Sun, we can use results for the Solar System:

$$P^2 = A^3$$
$$16 = A^3 \text{ (in AU)}$$
$$\boxed{A = 2.5\,\text{AU}}$$

(b) Note that the queston is asking how far the star is from *us*, not how far the planet is from the star.

$$F = \frac{L}{4\pi d^2}$$
$$d = sqrt\frac{L}{4\pi F}$$
$$d = sqrt\frac{3.85 \times 10^{26}\,\text{kg}\,\text{m}^2/\text{s}^3}{4\pi\,(8.04 \times 10^{-11}\,\text{kg/s}^3)}$$

If you are surprised by those flux units, note that W/m^2 is the same as kg/s^3, given that 1 W=1 J/s=1 $\text{kg}\,\text{m}^2/\text{s}^3$.

$$\boxed{d = 6.17 \times 10^{17}\,\text{m} = 20\,\text{pc}}$$

(c)

$$p = \frac{1}{20}'' = \boxed{0.05''}$$

(d) The maximum redshift is when the star is moving away from us (so the planet is moving towards us). The maximum blueshift will happen when the star is moving towards us (so the planet is moving away from us). This is half-way through the orbit of the planet, so this will happen in $\boxed{\text{2 years}}$.

(e) The angular diameter (in radians) is the physical diameter divided by the distance to the system. (See the picture for the solution to Problem 9.) Thus, we have:

$$A = \left(\frac{2.5\,\text{AU}}{20\,\text{pc}}\right)\left(\frac{1\,\text{pc}}{206,265\,\text{AU}}\right)$$

Note that we multiplied the quotient by 1 to make sure all the units come out right.

$$A = 6.06 \times 10^{-7}\,\text{radians} = 0.125''$$

This could be just resolved by the HST. **Note:** A number of you confused the parallax with the angular separation between the star and the planet. These are two very different things!

(f) You can set no limits. You would need the maximum value of the redshift or the blueshift of the star to figure out the mass of the planet. That would tell you how fast the star is moving. Remember that the star is moving (wobbling) as a result of the gravity of the planet. Greater gravity will make the star move more. We are given the distance, so the only way to have greater gravity due to the planet would be to increase the mass of the planet.