Astronomy 102, Fall 2003

## Homework Set 1 Solutions

1. There are a range of latitudes where, on at least one day of the year, the Sun will pass directly overhead (i.e. at the Zenith). Which latitudes of the Earth meet this condition? Does this happen in Nashville?

The sun varies between $+23.5^{\circ}$ and $-23.5^{\circ}$ declination. The declination at the zenith is the same as your latitude, thus the sun can be at the zenith if your latitude is $-23.5^{\circ}<l<+23.5^{\circ}$. This does not include Nashville (latitude $36^{\circ}$ ).
2. Chapter 2, Question 6: Sometimes artists paint the horns of the crescent moon pointing toward the horizon. Is this realistic? Explain.
Assuming that this is a night scene being painted, no, this is not realistic. There are two important points here.
First point: if the moon is crescent, it means that to an observer on the Earth, the angle between the moon and the Sun in the sky is fairly small (less than $90^{\circ}$ ). (If the angle were more than that, you'd have a gibbous or even full moon.) Thus, to be visible at night, the moon will either set not too long after the sun, or will rise not too long before the sun. A setting crescent moon will be over the western horizon just after the sun has gone beneath it, and a rising crescent moon will be over the eastern horizon just before the sun comes up over it
Second point: the crescent is on the side of the moon that is towards the sun, since the sun is what is lighting up that part of the moon. (Most of the lit side of the moon is away from us when the moon is a crescent; the difference in the angle to the Sun and moon lets us see a bit of the lit edge, which looks like a crescent to us.)

At a latitude similar to Nashville's in the northern hemisphere, the situation would look something like the picture below; the horns aren't pointing straight up, but they are pointing more up than down. At the equator, the sun would be setting straight down, and the moon would be (at most) at a $5.2^{\circ}$ angle to the Sun, often less. The "horns" would in fact be pointing almost straight up.


Note: Many of you made the argument that since the Moon's orbit is only tilted by 5 degrees relative to the Sun's orbit, the phases of the moon sweep between west and east on the moon; thus, the horns will always point primarily east or west, away from the Sun. This much is correct. However, many of you went on to say that because the horns point east or west, they can't point up or down. This is wrong! North/South is not the same as Up/Down. Consider a person on the Equator, facing his Western horizon. West on the sky is straight down, and East on the sky is straight up. In Nashville,
facing the Western horizon, West on the sky is down and to the right. Those of you doing the Sunset lab will hopefully observe the Sun setting at an angle - bearing in mind that the apparent motion of the Sun is to the west as it does this. If this confuses you, take another look at the Celestial Sphere animation available on the "Handouts" section of the course website.
3. Chapter 2, Question 13: The Moon's orbit is tilted by about $5^{\circ}$ relative to Earth's orbit around the Sun. What is the highest altitude in the sky that the Moon can reach as seen in Philadelphia (latitude 40 )?
There are two ways to do this. The first way is to recognize that the declination of the zenith is the same as our latitude, $40^{\circ}$. The altitude of the zenith is always $90^{\circ}$, at a right angle to the horizon. The highest declination the Sun reaches is $23^{\circ}$, which is $40^{\circ}-23^{\circ}=17^{\circ}$ less than the declination of our Zenith (when it's due south on the summer solstices). At this time, the Sun's altitude is $90^{\circ}-17^{\circ}=73^{\circ}$ (the altitude of the zenith minus the difference between the zenith and the Sun). If the angle between the Moon and Sun is $5^{\circ}$, then the Moon can be at most $5^{\circ}$ closer to the zenith than the Sun, and thus can be at the highest altitude of $78^{\circ}$.
The other way to do this is just to add the angles directly. First, recognize that at latitude $40^{\circ}$, the angle between the horizon and the celestial equator is $90^{\circ}-40^{\circ}=50^{\circ}$, as shown in the picture below. (The angle from horizon to zenith is $90^{\circ}$, so horizon-to-equator plus equator-to-zenith must be $90^{\circ}$.) Due south, that is the altitude of the equator. The Sun gets to a maximum angle of $23^{\circ}$ above the equator, or a maximum altitude of $50^{\circ}+23^{\circ}=73^{\circ}$. The Moon can be at most another $5^{\circ}$ away from the Sun's highest point, or $73^{\circ}+5^{\circ}=78^{\circ}$.
Many of you did this a third way, by considering the situation at the Equator, and then adding the difference between the Equator's latitude and Philly's latitude. While your chain of reasoning was fine, there was one statement many of you made which is not entirely correct. You started by saying that the zenith at the Equator is the celestial equator (correct), and then that the moon can be $23^{\circ}+5^{\circ}$ off of the equator (also correct), meaning that $118^{\circ}$ is the "highest" in the sky that the moon can be as viewed from the equator. Be careful here! The highest anything can be in the sky is at the zenith, which is always at an altitude of $90^{\circ}$. However, those of you who did it this way, assuming you kept track of what you really meant (farthest angle from the Southern horizon, rather than "highest in the sky"), were able to then get the right final correct answer. Think, however, what you would have answered for an observer at $10^{\circ}$ latitude. (Hint: the answer is just $90^{\circ}$ altitude.)
4. Chapter 2, Question 2: Assume that the Moon's orbit is circular. Suppose you are standing on the side of the Moon that faces Earth. How would Earth appear to move in the sky as the Moon made one revolution around Earth? How would the "phases of Earth" appear to you, as compared to the phases of the Moon as seen from Earth?
The moon rotates with the same period as its orbit around the Earth, thus the same side of the Moon faces the Earth at all times. This means that to an observer on the Moon, the Earth does not appear to move across the sky. However, the Earth may be seen rotating, since its rotational period ( 24 hours) is much shorter than the orbital period.
The Earth would show phases from the Moon, precisely for the same reason the Moon shows phases from the Earth. Depending on whether the Sun is behind you (and thus lighting the face of the Earth you can see), or on the far side of the Earth from you (and thus lighting the face of the Earth you can't see), you will see a "full Earth" or "new Earth". The phases of the Earth as seen from the Moon are exactly opposite the phases oft he Moon as seen from Earth:

5. Chapter 2, Question 9: What is the approximate time of day when you see the full Moon near the meridian? At what time is the first quarter (waxing) Moon on the eastern horizon? Use a sketch to help explain your answers.
Consider the following picture, looking "down" on the Earth/Moon system (so that we're looking down on the North pole of the Earth):


The full Moon happens when we can see the fully illuminated side of the Moon- meaning that when we're looking at the Moon, the Sun must be behind us, so that the face of the Moon lit by the Sun faces us. This means that the Sun is in the opposite direction of the sky from the Moon. When the Moon crosses the meridian, it is as high in the sky as it will get, so the Sun must be as low as it gets, which is the middle of the night. Thus, the full Moon will cross the meridian at midnight.
The first quarter (when the moon is waxing, or growing more full) will happen when the Moon is in the position shown in the picture. (As the moon continues in its orbit, it gets closer to the "full Moon" position - so it's waxing, or getting closer to full.) When anything is on the eastern horizon, it is rising (as things rise in the east and set in the west from our point of view on Earth). Given the rotation of the Earth as drawn in the diagram, the first quarter Moon is on the horizon for the guy standing in the middle of the day-lit side, and will only get higher in the sky as the Earth rotates. The guy is standing in the middle of the day-lit side of Earth, so it's noon when the first quarter moon is on the easter horizon and just rising.
6. In the northern hemisphere, June 21 is the longest day of the year and December 21 is the shortest day of the year. How long is the day (in hours) at the equator on June 21? Can you identify the shortest day in the year on the equator?
The day is 12 hours long every day on the equator. At the equator, the horizon cuts every circle of constant declination on the equator exactly in half (see Figure 2.8c in the text), so every object spends half of its time above the horizon, and half of the time below the horizon, regardless of its declination.

