Astronomy 102, Fall 2003

Homework Set 2 Solutions

1. The moon and all of the planets appear to go through phases, meaning that most of the time we see only a part of their surface illuminated. The Sun never goes through such phases. Explain why the Sun is different from the Moon and the planets.

This is ultimately because the sun is the source of illumination in the solar system. The moon and all of the planets are reflecting light from the Sun, and thus only the side facing the Sun is lit. Depending on the angle we see the other planets or the Moon, we see phases.

Many of you identified the fact that we are orbiting the Sun, and the Sun is at the center of the solar system, as a contributing factor. This is irrelevant to phases. Suppose the sun were a very low-mass but very bright point of light orbiting the Earth. As it went around the Earth, it would still create phases on the Moon and anything else it illuminated, even though it wasn't at the center of this hypothetical system.

A few of you incorrectly indicated that phases of the planets, or at least the Moon, have to do with another celestial body (e.g. the Earth) blocking the Sun's light. This is wrong! The Moon's phase is not due to the shadow of the Earth! Review the section in the text on the Moon's phases, and the "Moon Phase" animation on the handouts section of the course website.

2. Chapter 3, Question 11: Had Kepler lived on one of a group of planits orbiting a star three times as massive as our Sun, would he have deduced the same empirical laws? If not, how would they have been different? Explain.

Orbits are ellipses: this one would have been the same. All that's necessary for this is a "Keplerian" system, where all (or at least the vast majority) of the mass in the system is concentrated towards the center (inside the orbit of the innermost planet you're considering).

Equal areas in equal times: Likewise, this only requires a Keplerian system. The actual velocities might change, but the ratios of the velocities for an elliptical orbit of a given eccentricity would not.

 $P^2 = A^3$: This one would be modified. It would not work for P in years and A in astronomical units as we define them. Consider a planet at 1 AU orbiting in a circle. If the sun were more massive, that would mean greater gravitational force on a planet at a given distance, and therefore greater acceleration. Greater acceleration means that the velocity is changing faster. If you have a circlar orbit, the speed is constant, so all that can change about the velocity is the direction. Faster change in the velocity's direction means that it's getting around the circle in less time, so therefore P would be lower.

A number of you indicated that the eccentricities of the orbits would change (either to be higher or lower). This is not necessarily the case. Were you to take the planets in their current orbits *and* with their current orbital speeds, and then suddenly triple the mass of the Sun, yes, the planets would be pulled into elliptical orbits with higher eccentricities than their current orbits. However, there's nothing stopping planets from having circular orbits around a more massive star— they'd just have to be moving faster at a given distance from the star than the planets in our solar system.

3. Chapter 3, Question 17: Weight refers to the force of gravity acting on a mass. We often calculate the weight of an object by multiplying its mass by the local acceleration due to gravity. The value of gravitational acceleration on the surface of Mars is 0.39 times that on Earth. Assume your mass is 85kg. Then your weight on Earth is 833 N. What would your mass and wegith be on Mars? Give your answer in both pounds and Newtons.

Your mass on Mars would be 85kg; mass is a property of an object, and is unchanged by environment. To find your weight, we first need the value of gravitational acceleration on Mars:

$$g_M = (0.39)g = (0.39)\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right) = 3.822\frac{\mathrm{m}}{\mathrm{s}^2}$$

(A note on significant figures: even though we only have two significant figures in this problem, we're keeping four on this number since you always want to keep extra digits on numbers used in intermediate calculations.)

Now that we know this, we can calculate the weight:

$$\left(3.822\,\frac{\rm m}{{\rm s}^2}\right)(85\,{\rm kg})~=~325\,{\rm N}$$

(Strictly speaking, we only have two significant figures and thus should be quoting 320 N. If you gave more than three or four significant figures, I took off a point.)

Knowing the conversion between Newtons and Pounds, we can convert this:

$$(324.87 \,\mathrm{N}) \left(\frac{0.2248 \,\mathrm{lbs}}{1 \,\mathrm{N}}\right) = 73 \,\mathrm{lbs}$$

Notice again that I only quote the answer to two significant figures, but that I keep extra significant figures on the numbers used for intermediate calculations.

A few of you gave *mass* in pounds. Strictly speaking, pounds is a *weight* unit. When you convert kilograms to pounds (and you can look up that conversion), you're implicitly assuming that gravity is equal to the value at the surface of the Earth, and converting your weight in Newtons on Earth to pounds. (Note that an 85 kg person weighs about 187 pounds on earth.)

- 4. One of the most famous comets in the solar system is Comet Halley. comet. This comet returns to the inner solar system and is visible every 76 years; it's last apparition was in 1986. The comet, like other periodic comets, is in a highly eccentric elliptical orbit around the Sun.
 - (a) What is the semi-major axis of the orbit of Comet Halley?
 - (b) Comet Halley is typically only visible for several months out of the 76 years of its orbit. Use Kepler's laws to explain why.
 - (c) When should we expect to see Comet Halley again?
 - (a) The semi-major axis is the same as the average distance of the comet from the Sun, and is the A in $P^2 = A^3$. Knowing that P = 76 years, we can figure out A:

$$A = P^{2/3} = 76^{2/3} \,\mathrm{AU} = 18 \,AU$$

- (b) Kepler's "equal areas in equal time" law tells us that an object orbiting the Sun moves *faster* when it is closer to the Sun. 18 AU is a good long way out, at which distance it is difficult to see a comet. We're really only going to see the comet well when it is closer to 1 AU away from the Sun, where the Earth is. However, this is when the comet is moving the fastest, so it gets through this closer part of its orbit in much less time than it gets through the more distant part of its orbit. Thus, the comet is only close for a small fraction of the time it spends going around it's entire elliptical orbit.
- (c) 1986+76=2062. I don't know if I'll still be alive, but many of you will be.
- 5. You are standing on the surface of the Earth. Calculate the gravitational force on you (assume you mass 75kg if you don't know your mass) from (a) the Earth, (b) the Sun, (c) the Moon, (d) Mars, and (e) Jupiter. Assume that Mars and Jupiter are each about as close as they get to the Earth (0.5 AU and 4.2 AU respectively). Here are some more numbers you will need:
 - Radius of the Earth: 6400 km
 - Mass of the Earth: $6.0 \times 10^{24} \text{ kg}$
 - Distance from the Sun to the Earth: 1 AU
 - Mass of the Sun: 2.0×10^{30} kg

- Distance from the Earth to the Moon: 380,000 km
- Mass of the Moon: 7.4×10^{22} kg
- Mass of Mars: 6.4×10^{23} kg
- Mass of Jupiter: 1.9×10^{27} kg

(Remember to convert units to a consistent system to get answers that make sense!)

For comparison, calculate the gravitational force on you from a car (2000kg) that is 1m away from you. Think about this next time somebody tells you that the motions of the planets, or their position at your birth, may affect your character or your daily life...

The basic procedure here is the same in all cases. We use Newton's law of universal gravitation:

$$F = \frac{G M_1 M_2}{r^2}$$

where $G = 6.67 \times 10^{-11} \text{m}^3/\text{kg s}^2$, M_2 is your mass (75kg), M_1 is the mass of the object we're calculating the gravitational force due to, and r is the distance from you to that object. Make sure to convert all distances to meters before proceeding! $1 \text{AU}=1.5\times 10^{11} \text{ m}$, and of course 1 km=1,000 m. Once you have this, it's just a matter of cranking the numbers (and rounding them to about the right number of significant figures)....

- (a) Earth: F = 730 N. (This is your weight!)
- (b) Sun: F = 0.44 N.
- (c) Moon: F = 0.0026 N.
- (d) Mars: $F = 5.7 \times 10^{-7}$ N.
- (e) Jupiter: $F = 2.4 \times 10^{-5}$ N.

Notice that even though Mars is much closer, Jupiter has a stronger gravitational force on you. Notice also that the gravitational force of every object except for the Earth is tiny. (If the Sun's gravitational force were a significant fraction of the Earth's gravitational force, then you'd weigh a different amount at noon and midnight!)

For comparison, a car which is 1m away exerts a gravitational force of 1×10^{-5} N... nearly twenty times the force exerted on you by Mars. Yet, some people will tell you that the position of Mass at your birth makes a difference in your character and your fate. If you believe astrology, consider the order of magnitude of these numbers. If the position of Mars or Jupiter matters at your birth, the assuredly so does all of the various equipment in the hopsital room where you were born, never mind the weight and position of the doctors and the nurses.

In the questions I asked you to answer on the first day of class, a couple of you mentioned wanting to know the connection between astronomy and astrology. Today, the answer is as follows: Astronomy is a science. Astrology is nonsense. Historically, astrology was part of humanity's attempt to make sense of patterns in the sky. Today, we recognize that constellations are just chance alignments of stars, and it is our minds that project patterns (and, sometimes, the pictures of animals) onto them. Our knowledge of science has advanced to the point that we can perform real scientific studies and truly understand what is going on in the sky, without needing to resort to mythology. Indeed, the predictions of astrology itself have been tested scientifically— and it has been shown that there is absolutely nothing to them. Nonetheless, many today continue to believe in astrology, despite no reason to suppose it is true, and despite evidence to the contrary. When you see astrology being promoted, it is either just entertainment not to be taken seriously, ignorance, or fraud (deception of either self or others). (I would not call it "harmless" entertainment, since publishing astrology even as entertainment does end up promoting ignorance among those in the public who read it and think there might be something to it.)

You can find more information about astrology and why it is bogus in the list of references here:

http://www.astrosociety.org/education/resources/pseudobib.html#1