

Astronomy 102, Fall 2003
Homework Set 3 Solutions

1. *In class, I told you that if the Sun were a grain of sand and Proxima Centauri (the closest star to the Sun) were another grain of sand, they would be something 20 miles apart. Create a similar analogy for the Milky Way and Andromeda galaxies, only using frisbees or dinner plates to represent the disks of these Galaxies. Comment on what this says about how often you might expect to see galaxy/galaxy collisions in the Universe compared to star/star collisions in our Galaxy. (For the size of our Galaxy and the distance to the Andromeda Galaxy, refer to the Reading Assignment from September 17.)*

The disk of the Milky Way galaxy is about 100,000 light-years, or 0.1 million light-years, across. It is about 2.6 million light-years away from the Andromeda galaxy. If we model the disk of each galaxy as a dinner plate (call it 1 foot in diameter for simplicity), then the two dinner plates should be about 26 feet apart. This is easy to visualize; you can set down a couple of plates at either end of a long table, and you can see how far apart the galaxies are compared to their size. Galaxies are *much* closer to each other relative to their size than are stars compared to *their* size, and thus you'd expect to see galaxy/galaxy collisions far more often than you see star/star collisions. Indeed, if we look out into the Universe, we see quite a number of interacting galaxies; one example is on the title page of your textbook. We never see stars in the disk of our Galaxy run into each other, however. (We do see binary stars, but they are stars which have formed together, and have always been companions.)

2. Chapter 4, Question 14: *On a dark night, you notice that a distant lightbulb happens to be the same brightness as a firefly that is 5m away from you. If the lightbulb is a million times more luminous than the firefly, how far away is the lightbulb?*

Start by writing down what we know. Remember that brightness is flux, or how bright an object appears to you as an observer. Use the subscript B for quantities relevant to the lightbulb, and the subscript F for those of the firefly.

$$L_B = 10^6 L_F$$

$$F_B = F_F$$

$$d_F = 5 \text{ m}$$

$$F_B = \frac{L_B}{4\pi d_B^2}$$

$$F_F = \frac{L_F}{4\pi d_F^2}$$

Given the second line ($F_B = F_F$), we can set the last two lines equal to each other:

$$\frac{L_B}{4\pi d_B^2} = \frac{L_F}{4\pi d_F^2}$$

Multiply both sides by 4π :

$$\frac{L_B}{d_B^2} = \frac{L_F}{d_F^2}$$

and then finish solving it for d_B , the distance to the lightbulb.

$$d_B^2 = d_F^2 \left(\frac{L_B}{L_F} \right)$$

$$d_B = d_F \sqrt{\frac{L_B}{L_F}}$$

$$d_b = (5 \text{ m}) \sqrt{10^6}$$

$$\boxed{d_b = 5,000 \text{ m}}$$

3. An astronomer detects two yellowish stars which she believes have identical luminosities. Star A has a measured parallax of $0.125''$. Star B is detected to be 400.0 times dimmer than Star A. What is the distance to star B?

This problem is extremely similar to the previous problem, but many of you who did the previous one correctly had trouble with this one. Start by writing down what we know:

$$F_A = 400.0 F_B$$

$$L_A = L_B$$

$$F_A = \frac{L_A}{4\pi d_A^2}$$

$$F_B = \frac{L_B}{4\pi d_B^2}$$

We also know the parallax of star A, and can use this to figure out the distance to star A:

$$d_A = \frac{1}{p} = \frac{1}{0.125} \text{ pc} = 8 \text{ pc}$$

(This gives us the distance in pc because the parallax was in arcseconds, as $d = 1/p$ requires.)

What we want is d_B . Start again by substituting the third and fourth equations above into the first equation:

$$\frac{L_A}{4\pi d_A^2} = 400.0 \frac{L_B}{4\pi d_B^2}$$

Solve this for d_B :

$$d_B^2 = 400.0 \frac{L_B}{L_A} d_A^2$$

Since $L_B = L_A$, the fraction in the middle goes away:

$$d_B^2 = 400.0 d_A^2$$

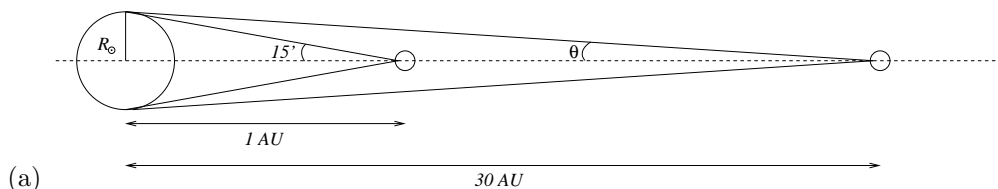
$$d_B = 20 d_A$$

$$\boxed{d_B = 160 \text{ pc}}$$

4. Chapter 4, Question 11 (skip b): *In our sky, the angular diameter of the Sun is about 30 minutes of arc. Now imagine that you have been transported to Neptune, 30 AU from the Sun.*

(a) *What would be the angular diameter of the Sun as seen in Neptune's sky?*

(c) *How bright would the Sun appear as compared to its brightness as seen from Earth?*



$$\left(\frac{R_{\odot}}{1 \text{ AU}} \right) = \tan 15'$$

Note that the $15'$ is there because it's *half* of the angular diameter of the Sun, to get it to work in the right triangle as drawn. It's tempting to use the small angle formula, but we have to remember that only works for radians:

$$15' \left(\frac{60''}{1'} \right) \left(\frac{1 \text{ rad}}{206265''} \right) = 0.00465 \text{ rad}$$

It's also important to keep at least an extra digit or two beyond the significant figures of the problem for intermediate calculations!

Having this, we can write:

$$\left(\frac{R_{\odot}}{1 \text{ AU}} \right) \approx 0.00465$$

We could solve this for R_{\odot} , but instead let's take a shortcut. We also know, from the other triangle in this problem:

$$\left(\frac{R_{\odot}}{30 \text{ AU}} \right) = \tan \theta \approx \theta$$

Divide the second equation by the first equation:

$$\frac{1/30 \text{ AU}}{1/1 \text{ AU}} = \frac{\theta}{0.00465}$$

$$\frac{1 \text{ AU}}{30 \text{ AU}} = \frac{\theta}{0.00465}$$

$$\theta = \frac{0.00465}{30} = 0.000155 \text{ radians}$$

It would be more interesting to have this in arcminutes, so convert it back:

$$0.000155 \text{ radians} \left(\frac{206265''}{1 \text{ radian}} \right) \left(\frac{1'}{60''} \right) = 0.53'$$

This is only *half* of the angular diameter, so the full angular diameter is $\boxed{1'}$.

- (b) Although we haven't discussed this, the human eye can resolve about $0.5'$. Thus, the Sun would *barely* be resolved into a disk to the naked eye, as opposed to just looking like a point.
- (c) How bright it looks is flux. If F_E is the flux as seen from the earth, and F_N is the flux as seen from Neptune, then we have:

$$F_E = \frac{L_{\odot}}{4\pi(1 \text{ AU})^2}$$

$$F_N = \frac{L_{\odot}}{4\pi(30 \text{ AU})^2}$$

Divide these two to get the ratio of fluxes:

$$\frac{F_E}{F_N} = \frac{L_{\odot}/(4\pi(1 \text{ AU})^2)}{L_{\odot}/(4\pi(30 \text{ AU})^2)}$$

Cancel out the stuff that appears on the top and the bottom:

$$\frac{F_E}{F_N} = \frac{(30 \text{ AU})^2}{(1 \text{ AU})^2} = 30^2$$

$$\boxed{\frac{F_E}{F_N} = 900}$$

The sun appears 900 times dimmer from Neptune than it does from Earth. Note that even though this is a lot dimmer, the Sun would still be the brightest star in Neptune's sky, by far.

A few of you tried to use $L = (\sigma T^4)(4\pi R^2)$ to show this. While you end up with a 900, what you have done doesn't make sense. Even though you've got a variable with length units (R) here, it's important to know what the variables mean when you use a given equation. This equation here tells you the luminosity of an object when you know the temperature and radius of that object. We aren't given the radius of the Earth or Neptune, we're given the distance between the Sun and each of those planets. What's more, we're not trying to figure out the Luminosity of either planet, which is what this equation would give you.

5. Chapter 4, Question 12: *Imagine you are riding in a car, tuned into 790 on the AM radio dial. This station is broadcasting at a frequency of 790 kilohertz (7.9×10^5 Hz). What is the wavelength of the radio signal? You switch to an FM station at 98.3 on your dial. This station is broadcasting at a frequency of 98.3 megahertz (9.83×10^7 Hz). What is the wavelength of this radio signal?*

$$\lambda f = c$$

$$\lambda = \frac{c}{f}$$

AM signal:

$$\lambda = \frac{3 \times 10^8 \text{ m s}^{-1}}{7.9 \times 10^5 \text{ s}^{-1}}$$

$$\lambda = 380 \text{ m}$$

FM signal:

$$\lambda = \frac{3 \times 10^8 \text{ m s}^{-1}}{9.83 \times 10^7 \text{ s}^{-1}}$$

$$\lambda = 3.05 \text{ m}$$