Astronomy 102, Fall 2003

Homework Set 4 Solutions

1. *How many radio photons (assume a wavelength of 1m) does it take to equal the energy of a single ultraviolet photon (assume a wavelength of 100nm, remembering that one nanometer (nm) is 10^{-9}m).*

$$\text{photon energy} = hf = \frac{hc}{\lambda}$$

where h is Planck's constant (6.626, c is the speed of light, and λ is the wavelength. Thus we have:

$$E_{\text{UV}} = \frac{hc}{10^{-7}\text{ m}}$$

$$E_{\text{Radio}} = \frac{hc}{1\text{ m}}$$

At this point, we could look up and plug in values of h and c to get these two energy values. Then, we'd divide E_{UV} by E_{Radio} to figure out how many Radio photons it takes to equal the energy of one UV photon. This is equivalent to solving the equation:

$$nE_{\text{Radio}} = E_{\text{UV}}$$

for n. Note, however, that we don't actually ever need to look up h and c, if we just solve it algebraically:

$$\frac{E_{\text{UV}}}{E_{\text{Radio}}} = \frac{hc/10^{-7}\text{ m}}{hc/1\text{ m}}$$

$$= \frac{1/10^{-7}}{1/1} = \frac{10^{7}}{1} = 10^{7}$$

The h and c cancel (divide out). It takes $\boxed{\text{10 million}}$ 1m radio photons to equal the energy of a single 100nm UV photon.

2. *You want to escape from the Solar System! Waaaah! You strap yourself into your rocket and blast off from the vicinity of the Earth. Assuming that you've already managed to escape from the Earth (but are still travelling along in the direction of Earth's orbit), and now only need to break free of the gravitational grip of the Sun, in which direction do you want to point your rocket so as to escape using the minimum possible amount of fuel? Do you want to point it towards the Sun, away from the Sun, in the same direction as Earth's motion about it's orbit, in the direction opposite Earth's motion, or in some direction between one of these four "cardinal" directions? Justify your answer in terms of energy; your goal is to get a very large distance away from the Sun.*

You want to point along the direction of Earth's orbit when you fire your rocket.

In terms of energy: you want to get very far from the Sun. As two objects get farther away, they have *more* potential energy. Thus, your eventual goal is to greatly increase your potential energy.

By firing your rockets, though, you don't immediately change your potential energy (which requires changing your position), but only your speed. By changing your speed, you can change your *kinetic* energy. When you're coasting in an orbit about a star, total energy is conserved; depending on where you are if your orbit is elliptical (or unbound), you might have more or less kinetic energy, or more or less potential energy, but your total energy will be constant.

Thus, what you need to do here is increase your total energy; you will give yourself a boost in kinetic energy, and then as you move out away from the Sun that kinetic energy will be converted into potential energy. The most efficient way to increase your kinetic energy is to increase your speed, so you want to fire your rockets to add to your velocity in the direction you are already moving. Any other direction, and you're "wasting" some of your fuel in turning— i.e., some of the acceleration goes into changing the direction of your velocity.

3. *Atomic emission "lines" as described in class always happen at exactly one wavelength: the wavelength where photons have the same energy as a specific atomic transition. In fact, atronomers usually observe "broadened" lines from astrophysical sources, where a line at a given wavelength is spread into nearby wavelengths. One very common mechanism for this broadening is called "Doppler broadening": if a gas cloud is turbulent, with atoms moving about randomly, some of the atoms in the cloud will be moving towards you, some away from you. There will be a blueshift or a redshift of the light emitted by atmos moving towards or away from you, and as the light from all of the atoms is added together you get a smeared out (or broadened) line. Consider the 6563? line of Hydrogen. Suppose from a nebula it is observed to be spread from 6560-6566?. What range of speeds towards/away from you would you conclude the atoms in this nebula are moving, if you interpret this broadening as a doppler broadening? (Give your answer in km/s.)*

The most negative velocity you will get is when the gas is coming towards you and is therefore blueshifted to the lowest observed wavelength:

$$\begin{aligned}
\frac{v_{\min}}{c} &= \frac{6560\,\text{Å} - 6563\,\text{Å}}{6563\,\text{Å}} \\
v_{\min} &= (-0.000457)(3.00 \times 10^8\,\frac{\text{m}}{\text{s}} \\
v_{\min} &= -140,000\,\frac{\text{m}}{\text{s}} \\
v_{\min} &= -140\,\text{km/s}
\end{aligned}$$

Similarly, the highest velocity you will get is when the gas is receeding and therefore redshifted to the lowest observed wavelength:

$$\begin{aligned}
\frac{v_{\min}}{c} &= \frac{6566\,\text{Å} - 6563\,\text{Å}}{6563\,\text{Å}} \\
v_{\min} &= (+0.000457)(3.00 \times 10^8\,\frac{\text{m}}{\text{s}}) \\
v_{\min} &= 140,000\,\frac{\text{m}}{\text{s}} \\
v_{\min} &= 140\,\text{km/s}
\end{aligned}$$

Thus, the range is $\boxed{\text{-140 to 140 km/s}}$, or equivalently everything between 140 km/s approaching and 140 km/s receeding.

Note that the "everything between" is important! A number of you said "-140 or 140 km/s" or "either -140 or 140 km/s". This is not right! In that case, you would see a split line, with two very narrow lines at 6560Åand 6563Å. To get a *broadened* line, you need a continuous range of velocities.

A note on significant figures: Really the answer to this problem should only have *one* significant figure! I only took off a point if you reported more than three. Why only one, when it looks like the initial numbers are given to four sig figs? Go back and look at the Math Review again. Notice that you are subtracting two numbers, 6560-6563. That subtraction is only significant to the *ones* place, and that's the point at which your number of significant figures gets reduced to one.

4. *Is it easier to find planets around stars whose orbits are "face-on" (i.e. you are looking "down" on the plane of the orbit) or "edge-on" (i.e. you are looking at "the side" of the plane of the orbit)? Why?*

We talked about this in class. Most extrasolar planets have been found by observing the Doppler shift in the star from its reflex motion due to the pull of a planet orbiting it. When you are observing a system face-on, all of that reflex motion is in the "plane of the sky", and none of it is towards or away from you. When you're observing it edge-on, at times all of the motion is either towards or away from you, and you'll see the maximum possible Doppler shift.

5. *The 6563Åline of Hydrogen from the Andromeda galaxy is observed at 6556Å.*

(a) *Is the Andromeda galaxy moving towards us or away from us?*

(b) *How fast (at what speed) is the Andromeda galaxy moving along the line of sight?*

(c) *If the Milky Way and Andromeda galaxies are 2.6 million light-years apart, and you assume that there is no tangential velocity (velocity perpendicular to the line of sight), how long will it be before the two galaxies either collide, or until the distance between them doubles? (Indicate which will happen.)*

(a) It is moving $\boxed{\text{towards}}$ us. We observe the line at a lower wavelength, which is bluer light. A blueshift happens when something is approaching you.

(b)

$$\frac{v}{c} = \frac{\lambda_{\rm obs} - \lambda_0}{\lambda_0}$$

$$v = \left(\frac{6556\text{Å} - 6563\text{Å}}{6563\text{Å}}\right)(3.00 \times 10^8 \text{ m/s})$$

$$v = (-0.001)(3 \times 10^5 \text{ km/s})$$

$$\boxed{v = 300 \text{ km/s towards us}}$$

(c) The distance between the two galaxies is about 2.6 million light years. You can lookup that 1 lyr=9.5×10^{12} km. Then:

$$d = v\,t$$

$$t = \frac{d}{v}$$

$$t = \frac{(2.6 \times 10^6 \,\text{lyr})(9.5 \times 10^{12} \,\text{km}\,\text{lyr}^{-1})}{300 \,\text{km}\,\text{s}^{-1}}$$

$$t = (8.23 \times 10^{16}\,\text{s})\left(\frac{1\,\text{yr}}{3.16 \times 10^7\,\text{s}}\right)$$

$$t = 2.6 \times 10^9\, yr$$

So the two galaxies will collide, if they're coming right towards each other, in $\boxed{\text{2.6 billion years}}$.