Astronomy 102, Fall 2003

Homework Set 5 Solutions and Commentary

1. Chapter 19, Question 5. *Why is the Earth not expanding together with the rest of the Universe?*

Because it's gravity holds it together. Just as the tidal forces from the Sun as the Earth orbits it would tend to rip the Earth apart, the *locally* stronger gravity of the Earth holds it together. The same is true for galaxies. Yes, there is Dark Energy in galaxies which would tend to try to push everything away from each other, but within a galaxy matter is much denser than it is in the Universe as a whole, and that local enhancement of gravity holds it together. (This was also discussed at length in class on October 22.)

A few of you noted correctly that if the Earth were expanding right along with the Universe, we wouldn't notice the Universe's expansion! That is, if everything were scaling up together— including our planet, us, and, significantly, our yardsticks— we couldn't use those yardsticks to notice that the wavelength of light had gotten longer, and that galaxies were moving away from each other.

Note: A few of you said that "gravity" is stronger than the expansion of the Universe. In fact, it is gravity that is making that expansion accelerated! Dark energy is so bizarre that its gravitational effect is *negative*, i.e. repulsive. But this all comes directly from Einstein's theory of General Relativity, which described gravity.

Note: Although this is not relevant to this problem, galaxies *do* change. This is not directly due to the expansion of the Universe, but rather due to their internal properties. Consider yourself: you change over your life. (For example, you grow.) This has nothing to do with the Earth's rotation or the Earth's orbit around the Sun, but it does happen thanks to your own internal dynamics. Our galaxy is also growing, as it eats things like the Sagittarius Dwarf Galaxy.

2. Chapter 19, Question 15 *Suppose we observer two galaxies, one at a distance of 35 Mly with a radial velocity of 580 km/s, and another at a distance of 1,100 Mly with a radial velocity of 25,400 km/s.*

- ***(a)*** *Calculate the Hubble constant for each of these two observations.*
- ***(b)*** *Which of the two calculations would you consider to be more trustworthy? Why?*
- ***(c)*** *Estimate the peculiar velocity of the closer galaxy.*
- ***(d)*** *If the more distant galaxy had this same peculiar velocity, how would that change your calculated value of the Hubble constant?*

(a) In the "local Universe" approximation where we pretend that cosmological redshifts are Doppler shifts and it's a good approximation to pretend that galaxies which are getting more distant from us due to the expansion of space are flying away from us at a given velocity, we can use the Hubble law.

$$v = H_0\, d$$
$$H_0 = \frac{v}{d}$$

$$\text{Closer galaxy}: H_0 = \frac{580\,\mathrm{km/s}}{35\,\mathrm{Mly}} = \boxed{17\,\frac{\mathrm{km}}{\mathrm{s\,Mly}}}$$

$$\text{Farther galaxy}: H_0 = \frac{25400\,\mathrm{km/s}}{1100\,\mathrm{Mly}} = \boxed{23\,\frac{\mathrm{km}}{\mathrm{s\,Mly}}}$$

(b) The calculation from the more distant galaxy. Reason: peculiar velocities are always a few hundred km/s. They're random, so they could be anywhere from minus a few hundred to plus a few hundred. Potentially, this could be a large fraction of the 580 km/s of the closer galaxy. However, it will be a small fraction of the 25,400 km/s of the more distant galaxy.

A few of you said that the closer one should be more reliable because closer galaxies tend to be brighter, and therefore the measurements are less likely to suffer from observational errors. While this is true, the problem presents numbers to (at least approximately) equivalent significant figures in both cases. It may well have taken a lot more telescope time and effort to get the numbers on the more distant galaxy, but you have them. The peculiar velocity issue, however, is an *intrinsic* issue that perfect observations cannot get around. You will always have to deal with the reality of galaxies moving about in the Universe even if you have amazing data.

A number of you said that the more distant galaxy was more reliable because it agreed with the consensus of the scientific community. This is usually not a good reason to believe one method more reliable than another; unfortunately, even scientific researchers fall into the trap of believing that it is. Yes, frequently a good way to test a new method is to ensure that it can reproduce a well-known and well-tested result. However, if you really trust your new method, and it gives results different from the "consensus" belief, then the right thing to do is to believe and publish your result, and let the scientific community grapple with it. Einstein fell into this trap when he added the cosmological constant to be able to (sort of) predict the static Universe that the scientific community accepted. He could have "discovered" the expanding Universe theoretically before it was truly discovered observationally. (On the flip side, it can be just as bad to blindly trust your new and sexy method over a well-established result. When the Hubble Space Telescope was first launched, it had a bad primary mirror. It turns out that two tests of the mirror figure had been done. A more expensive, more sophisticated test said its figure was good, while an older and less precise test showed that there were problems. The engineers chose to trust the more sophisticated test and just dismiss the older method. It turns out that the newer test had been slightly miscalibrated, and the telescope was launched with a bad mirror. True progress of science is filled with pitfalls in all directions, and it is our obligation to do as best we can given our abilities and our situations.)

(c) A lot of you on this problem complained (either in office hours or in writing on the homework) that you couldn't "find the formula for peculiar velocity". Unfortunately, this sort of thinking represents the wrong approach not only to this problem, but to mathematics as applied to science in general. Yes, frequently when solving problems (either in homework or in true research) we progress by finding a formula somebody else has figured out for us and plugging in the right values. However, somebody else had to figure out those formulae. More fundamentally, these formulae aren't just magical recipes; they really do represent something. The mathematics is simply a language for describing reality. If you understand what something *is* well enough, and it's something whose mathematical description is tractable given the level of your mastery of mathematics, then you should be able to express it mathematically without being given the formula. Otherwise, you do not understand it as well as you have convinced yourself that you do.

What does this have to do with peculiar velocities? We didn't discuss them in class, but they were covered in the reading assignment. There is a fair amount of text describing it, but the key sentence fragment is: "Departures from a smooth Hubble flow, referred to as **peculiar velocities**...." Together with the description of the motion of a river and things that aren't going along with the flow, this tells you that a peculiar velocity is the difference between the actual, measured velocity of a galaxy and what its velocity would be if it were just moving along with the flow. Given that, the "formula" for peculiar velocity is just subtraction!

$$v_{\text{total}} = v_{\text{exp}} + v_{\text{pec}}$$

where v_{total} is the total velocity of a galaxy, v_{exp} is the "expansion velocity" (that velocity it would have if it were going along perfectly with the Hubble flow), and v_{pec} is the peculiar velocity.

How do we estimate the peculiar velocity of the closer galaxy? Well, we have a more reliable Hubble constant from the more distant galaxy (see (b)), so we can use that to calculate v_{exp} for the closer galaxy:

$$v_{\text{exp}} = H_0\, d = \left(23\,\frac{\text{km/s}}{\text{Mly}}\right)(35\,\text{Mly})$$

$$v_{\text{exp}} = 805\,\text{km/s}$$

Subtract this from v_{total} (580 km/s) to figure out that $\boxed{v_{\text{pec}} = -220\,\text{km/s}}$, that is, 220 km/s toward us.

(d) Suppose that 220 km/s of the 25,400 km/s we observed for the more distant galaxy were due to peculiar velocity. If this were the case, how wrong is our estimate of the Hubble constant? In this hypothetical case, the v_{exp} for the more distant galaxy would really be 25,620 km/s (although really we only know this to three significant figures). We would then get:

$$H_0 = \frac{v}{d} = \frac{25620\,\text{km/s}}{1100\,\text{Mly}}$$

$$H_0 = 23.3\,\frac{\text{km/s}}{\text{Mly}}$$

This is only different from our previous estimate by about 1that we've only really got two significant figures in the estimate (as 1,100 Mly only has two significant figures), the difference is smaller than the precision of our measurement. This is a specific illustration of why, given that all galaxies will tend to have peculiar velocities of a few hundred km/s, more distant galaxies give you a more reliable estimate of the Hubble constant.

One or two of you noted in (b) that the more distant galaxy would suffer from the fact that we're looking back in time, so the Hubble constant we are measuring is not *today's* expansion rate, but rather something more complicated. (It's not exactly the expansion rate back then either, it turns out.) While this is strictly correct, the expansion rate hasn't changed a whole heck of a lot in the last billion years (which about is how long it would take light to go 1,100 Mly), so that won't throw off an estimate of H_0 very much.

3. Chapter 19, Question 11: *Hubble time* $(1/H_0)$ *represents the age of a universe that has been expanding at a constant rate since the Big Bang. Assuming a* H_0 *value of 23 km/s/Mly and a constant rate of expansion, calculate the age of the Universe in years. Note that one year* $= 3.16 \times 10^7$ *s and one light-year* $= 9.46 \times 10^{12}$ *km.*

$$t_{\text{Hub}} = \frac{1}{H_0} = \frac{\text{s Mly}}{23\text{ km}}$$

To convert this to years, multiply it by 1 several times:

$$\left(\frac{\text{s Mly}}{23\text{ km}}\right)\left(\frac{10^6\text{ ly}}{\text{Mly}}\right)\left(\frac{9.46\times 10^{12}\text{ km}}{\text{km}}\right)\left(\frac{1\text{ yr}}{3.16\times 10^7\text{ s}}\right)$$
$$= \boxed{1.3\times 10^{10}\text{ yr} = 13\text{ billion years}}$$

4. Think about gravity vs. motion. If you want to throw a ball to a certain height above the ground, you will have to throw it harder on the Earth than you would on the Moon. Equivalently, it would need to start *faster* moving away from Earth in order to overcome enough of Earth's gravity in order to reach a certain hight than it would need in order to overcome the Moon's weaker gravity.

Similarly, if you see two clusters of a given size— that is, galaxies have been able to get "thrown" a certain distance away from the center of the cluster (which is the center of the cluster's gravitational well), the cluster with *more* mass (and therefore a deeper gravitational well) will need to have galaxies thrown faster. The velocity dispersion is the range of velocities you observe. In both clusters, you will see some galaxies with zero velocity, just as when you throw a ball into the air it briefly has no vertical velocity right as it reaches the top of its arc. In the cluster with more mass, though, this tells you that you'd see galaxies with higher velocities than the cluster with less mass. Thus, a signature of dark matter in elliptical galaxies or in clusters of galaxies would be a *higher* velocity dispersion than you would expect just from counting the mass that you can see (i.e. the stars and gas that we can detect with our telescopes).

5. Chapter 19, Question 14: *The rest wavelength of the Lyα line of hydrogen is in the extreme ultraviolet region of the spectrum at 0.1216 microns.*

(a) *What would be the wavelength of this line in the spectrum of a quasar with a redshift* $z = 5.82$

(b) *In what region of the spectrum would this red-shifted line be located?*

(a)

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}}$$

$$z = \frac{\lambda_{\rm obs}}{\lambda_{\rm em}} - \frac{\lambda_{\rm em}}{\lambda_{\rm em}} = \frac{\lambda_{\rm obs}}{\lambda_{\rm em}} - 1$$

$$1 + z = \frac{\lambda_{\rm obs}}{\lambda_{\rm em}}$$

$$\lambda_{\rm obs} = (1+z)\,\lambda_{\rm em}$$

$$\lambda_{\rm obs} = (1+5.82)\,(0.1216\,\mu{\rm m})$$

$$\boxed{\lambda_{\rm obs} = 0.8293\,\mu{\rm m} = 8293\,\text{Å}}$$

(b) This is just barely off of the red end of the visual range of the spectrum and in the near-infrared.

6. *When we look at very distant galaxies, we are looking at them as they were far in the past, because the light from them took time to reach us. Even though we're looking at distant galaxies, we talk about "looking at the earlier Universe", as if what we saw applied to the history of galaxies near us. Why do we think we can do this? What observational evidence supports these assumptions?*

We believe we can do this because of the cosmological principle. That is, if our galaxy isn't a special place in the Universe, then it will have gone (generally) through the same sort of evolution that very distant galaxies will have. Therefore, by looking at the past of very distant galaxies, we should be looking at something very similar to the past of our own galaxy. We won't see exactly our own galaxy's past, but we will see the sorts of things that would have happened.

What supports this assumption? There are observations that indicate the Universe is homogeneous and isotropic, that is, the same everywhere, and the same in all directions. We see the same distribution of galaxies in all directions. Perhaps the strongest piece of information is the smoothness of the Cosmic Microwave Background. Everywhere, this afterglow of creation, this left-over bit from when the Universe was a dense, hot plasma, shows almost exactly the same temperature, and we see the same tiny fluctuations in every direction. This is strong evidence that supports the cosmological principle, that tells us that the Universe is homogeneous and isotropic.