Astronomy 102, Fall 2003

Homework Set 7 Solutions and Commentary

1. Chapter 12, Question 5: *Albireo, a star in the constelaltion of Cygnus, is a binary system whose components are easily separated in a small amatuer telescope. Viewers describe the brighter star as "golden" and the fainter one as "sapphire blue".*

(a) What does this tell you about the relative temperatures of the two stars?

(b) What does it tell you about their respective sizes?

(a) The blue star is hotted. For thermally emitting bodies, bluer colors indicate hotter objects.

(b) The yellow star must be larger. A hotter star of the same size as a cooler star will be more luminous; a cooler star must be bigger to be more luminous than a hotter star. *Note:* a few of you said we couldn't tell, because it could just be that the golden star was closer. However, you are told that this is a binary system. (In other words, it's not just a chance alignment, but an actual physical system of two stars orbiting each other.) That means that the physical separation between the two stars is a lot less than the distance from us to the star, so the two stars are going to be at approximately the same distance.

2. *Look at the H-R diagram on the lower-left corner of Page 325 in your text. Consider a main-sequence star which is 10 times as massive as the Sun.*

(a) Would this star be bluer or redder than the Sun? Why?

(b) Look up the temperature and luminosity of this star on the H-R diagram. Calculate the radius of the star (in units of Solar Radii) (this is the same as the ratio of this star's radius to the Sun's radius). Now estimate the star's radius based on where it falls between the diagonal constant-radius lines on the diagram. How do your two values compare?

(c) Suppose this star is one of the faintest ones you can regularly see in lab? about 1012 (one trillion) times fainter than the Sun. How far away is this star?

(a) Bluer. Clearly, on the diagram, this star has a higher temperature than the Sun, so it will be bluer in color.

(b) On the diagram, it looks like we have about $T = 21,000\,\mathrm{K}$ and $L = 10^4\,L_\odot$. From this, we can figure out the size:

$$L = 4\,\pi\,R^2\,\sigma\,T^4$$

$$R = \sqrt{\frac{L}{4\,\pi\,\sigma\,T^4}} = \sqrt{\frac{(10^4)(3.85\times10^{26}\,\mathrm{W})}{4\pi\,(5.67\times10^{-8}\,\frac{\mathrm{W}}{\mathrm{m^2K^4}})(21,000\,\mathrm{K})^4}}$$

$$R = 5.3\times10^9\,m\left(\frac{1\,R_\odot}{6.96\times10^8}\right) = \boxed{8\,R_\odot}$$

(This answer is only good to one significant figure, as that's probably as well as you were able to interpolate the values on the figure). This is plausibly similar to the value you find from the size markings on the figure. (Anything between 5 and 10 $R_\odot$ is plausibly close enough given how accurate most of us are able to interpolate on this figure.)

The common error of just plugging in $L = 10^4$ without worrying about units was discussed in class.

(c) We have $F_\odot = 10^{12}\,F_*$, that is, the Sun is a trillion times brighter than the star. We also know $L_* = 10^4 L_\odot$ from the previous part of this problem.

$$f = \frac{L}{4\,\pi\,d^2}$$

$$d = \sqrt{\frac{L}{4\,\pi\,F}}$$

This applies for both the Sun and for this star. Divide the two in order to figure out the distance ratio (we *know* the distance to the Sun is 1 AU):

$$\frac{d_*}{d_\odot} = \frac{\sqrt{\frac{L_*}{4\,\pi\,F_*}}}{\sqrt{\frac{L_\odot}{4\,\pi\,F_\odot}}}$$

$$\frac{d_*}{d_\odot} = \sqrt{\frac{(L_*)(4\,\pi\,F_\odot)}{(L_\odot)(4\,\pi\,F_*)}}$$

$$d_* = \sqrt{\left(\frac{L_*}{L_\odot}\right)\left(\frac{F_\odot}{F_*}\right)}\; d_\odot$$

$$d_* = \sqrt{(10^4)(10^{12})}\;(1\,\mathrm{AU})$$

$$\boxed{d_* = 10^8\,\mathrm{AU} = 500\,\mathrm{pc} = 1.5\times10^{19}\,\mathrm{m}}$$

3. *Again consider the 10 solar-mass star of the previous problem. Assume that the amount of fuel available for producing energy is proportional to the mass of the star? that is, a star that is twice as massive has twice as much fuel to burn, and thus can produce twice as much energy over its lifetime. Given the relative mass of this star to the Sun, and the relative luminosity of this star and the Sun, how long will this star be able to shine? (The Sun will shine for about 10 billion years.)*

This star has 10 times the fuel of the Sun. However, it's using the fuel up 10,000 times as fast. Thus, it won't live as long as the sun. If it were using up fuel at the same rate as the Sun, it would live 10 times longer (10^{11} years), thanks to its extra fuel. However, with the higher rate of usage, the star will only live $10^{11}/10^4 = 10^7$ years, or 10 million years.

This simple estimate based on simple assumptions turns out not to be too terribly far off as an estimate of the lifetime of a star of this mass.

4. Chapter 12, Question 14: Sirius is 22 times more luminous than the Sun, and Polaris (the "Pole Star") is 2,350 times more luminous than the Sun. Sirius appears 23 times brighter than Polaris. What is the distance of Polaris in light-years?

Note: You need the answer of Question 13 to do this! That tells you that Sirius has a parallax of $0.379''$, so you can figure out that Sirius is at a distance of:

$$d_S = \frac{1}{0.379}\,\mathrm{pc} = 2.64\,\mathrm{pc}$$

Given that, we can now answer the question of the distance to Polaris by comparing its flux and luminoisty to that of Sirius:

$$F = \frac{L}{4\,\pi\,d^2}$$

$$d = \sqrt{\frac{L}{4\,\pi\,F}}$$

$$\frac{d_P}{d_S} = \frac{\sqrt{\frac{L_P}{4\,\pi\,F_P}}}{\sqrt{\frac{L_S}{4\,\pi\,F_S}}}$$

$$\frac{d_P}{d_S} = \sqrt{\frac{(L_P)(4\,\pi\,F_S)}{(L_S)(4\,\pi\,F_P)}}$$

$$
\begin{aligned}
\frac{d_P}{d_S} &= \sqrt{\left(\frac{L_P}{L_S}\right)\left(\frac{F_S}{F_P}\right)} \\
d_P &= \sqrt{\left(\frac{2350\,L_\odot}{22\,L_\odot}\right)(23)\,d_S} \\
d_P &= \sqrt{2460}\;(2.64\,\mathrm{pc}) \\
\end{aligned}
$$

$$\boxed{d_P \;=\; 130\,\mathrm{pc}}$$

5. Chapter 12, Question 16: *The Sun is about 16 trillion (*1.6×10^{13}*) times brighter than the faintest stars visible to the naked eye.*

(a) How far away (in AU) would an identical solar-type star be if it were just abrely visible to the naked eye?

(b) What would be its distance in light-years?

(a) See the flux/luminosity/distance math in the previous problem, and use that result:

$$\frac{d_*}{d_\odot} = \sqrt{\left(\frac{L_*}{L_\odot}\right)\left(\frac{F_\odot}{F_*}\right)}$$

Here, we have $L_* = L_\odot$, so:

$$d_* = \sqrt{1.6 \times 10^13}\;(1\,\mathrm{AU})$$

$$\boxed{d_* \;=\; 4.0 \times 10^6\,\mathrm{AU}}$$

(That's 4 million AU.)

(b)

$$(4.0 \times 10^6\,\mathrm{AU})\left(\frac{1\,\mathrm{pc}}{206265\,\mathrm{AU}}\right)\left(\frac{3.26\,\mathrm{lyr}}{1\,\mathrm{pc}}\right)$$

$$\boxed{63\,\mathrm{lyr}}$$