1. Chapter 13, Question 12: *Assume the Sun has been producing energy at a constant rate over its lifetime of 4.5 billion years $(1.4 \times 10^{17}$ s).*

(a) *How much mass has it lost creating energy over its lifetime?*

(b) *The present mass of the Sun is 2×10^{30} kg. What fraction of its present mass has been converted into energy over the lifetime of the Sun?*

(a) The luminosity of the Sun is 3.85×10^{26} W (where 1 W is 1 J/s). All of this comes from the conversion of mass into energy via fusion in the core. To produce this much energy, the Sun must use up an equivalent amount of mass each second:

$$E = mc^2$$

$$m = \frac{E}{c^2}$$

Remembering that a 1 Joule is $1 \text{ kg m}^2/\text{s}^2$:

$$m = \frac{3.85 \times 10^{26} \text{ J}}{(3 \times 10^8 \text{ m s}^{-1})^2}$$

$$m = 4.28 \times 10^9 \text{ kg}$$

This is the amount of mass used up *each second*. Over the lifetime of the Sun to date, then, the total mass used is:

$$m_{\text{used}} = (4.28 \times 10^9 \, \frac{\text{kg}}{\text{s}})(1.4 \times 10^{17} \text{ s})$$

$$\boxed{m_{\text{used}} = 6.0 \times 10^{26} \text{ kg}}$$

(b) To figure out what fraction of the Sun's mass has been converted to energy so far, just divide the mass used by the total mass of the Sun:

$$f = \frac{6.0 \times 10^{26} \text{ kg}}{2.0 \times 10^{30} \text{ kg}}$$

$$\boxed{f = 0.00030 = 3.0 \times 10^{-4}}$$

(This is 0.03% of the Sun's mass.) This answer is plausible; we know that only a small fraction of the mass of the four Hydrogen nuclei that get fused into a Helium nucleus is converted to energy, and the Sun has definitely not fused all of its Hydrogen, so we only expect a small fraction of the Sun's mass to have been used up. (Some of you had a fraction greater than one, which should have alarmed you.)

2. *Right now the Sun is a main-sequence star. Later in its life, it will become a red giant: it's luminosity will go up hundreds or thousands of times, it will be many (between ten and a hundred) times larger, and it will be a bit cooler.*

   (a) Will the rate at which the Sun converts some of its mass to energy be higher or lower when it is a red giant as compared to now? Why?

   (b) Do you expect that the pressure near the surface, but inside the Sun, will be higher when the Sun is a red giant than it is now? Why?

   (c) Do you expect the solar wind to be more intense (i.e. more particles coming off of the Sun per second) when it is a red giant or now?

(a) The rate will go up. We know this because the luminosity goes up; all of that energy comes from the conversion of mass to energy in the core. If the luminosity goes up, the rate of conversion of mass must go up. (The reason fusion is faster in the core is that it has become compressed relative to the core of the main sequence star; the hotter and denser core allows for faster fusion.)

Note: a few of you in your argument mentioned that the sun would be "more massive" as a red giant. This is incorrect! The Sun will *not* be more massive; it doesn't pull in any extra matter from anywhere. It *will* be bigger (larger volume), but will have at most the mass that it's got now. (When it starts out as a red giant, it will only be very slightly less massive than it is now, due to the slight mass loss from fusion (see problem 1) and from the currently meager solar wind.)

(b) The pressure will in fact be *lower*. You get higher pressures from higher density and higher temperature, but both are lower near the surface of a red giant. You know the temperature is lower because the star is red; redder in color means lower surface temperature. You know the density is lower because the star is so much bigger; the same mass spread over a larger volume gives you a lower density. (Many of you mentioned only temperature or density; you can be sure the pressure is lower in the red giant, because you know both are lower.)

A second way to argue this would be to use the knowledge that the balance of gravity and pressure is what holds the star up. The star has the same mass as a red giant as it did as a main sequence star, but is much bigger. Thus, the outer parts of the star are much farther from the center of mass, so the force of gravity will be less. Therefore, you need a lower pressure to counteract that gravity. (Initially, when the main sequence star uses up its fuel and its core contracts, you get more pressure near the core of the star. This causes the envelope to puff out, and once it settles down you have lower pressure near the surface.)

Note that the *core* of the star is in fact much denser when it's a red giant than when it's a main sequence star! However, the envelope of the star (which includes the part just below the surface) is much less dense.

(c) You would expect the stellar wind to be *higher*. The surface gravity is much lower (same mass, but bigger radius, so the force of gravity at the surface will be much smaller), so particles aren't held as tightly. Only the fastest particles tend to escape; the bar for escape is lower once the gravity is lower.

Note: many of you confused the light coming off of the star (that's what we measure with telescopes and what we're talking about when we use the word luminosity) with the stellar wind (particles coming off of the surface of the star). Just because the luminosity is higher (more energy is being generated), it necessarily doesn't mean that rate that particles are shed from the surface is higher.

3. *Consider a two large clouds of gas with solar abundances (i.e. mostly Hydrogen and Helium, with just a little bit of other things mixed in). Take one cloud of this gas, and make sun-like stars out of it. Let the sun-like stars live their main sequence lifetimes, and then (somehow, magically) disperse those sun like stars back into a cloud of gas. Qualitatively, how will the relative elemental abundances in the cloud which was processed through sun-like stars compare to the cloud which just remained a large, quiet cloud of gas?*

The cloud that was processed through main-sequence stars will have a *higher* Helium-to-Hydrogen ratio than the cloud that just sat there and did nothing. On the main sequence, stars use up Hydrogen to make Helium, so the abundance of Helium will go up while the abundance of Hydrogen goes down.

Note: A few of you said that the Helium abundance would be "much" higher. I didn't take off points for this, but that's not really quite right. The Sun will only use up a fraction of it's Hydrogen during it's main sequence lifetime: only that at the core. The envelope will remain mainly Hydrogen, and the Sun will still mostly be Hydrogen once it's a red giant.

4. Chapter 15, Question 13 (the one I intended to post!): *A white dwarf has a density of approximately 10^9 kg/m³. Earth has an average density of 5,500 kg/m³ and a diameter of 12,700 km. If Earth were compressed to the same density as a white dwarf, how large would it be?*

The volume of the Earth is:

$$V_E = \frac{4}{3}\pi R_E{}^3 = \frac{4}{3}\pi (6.35 \times 10^6 \text{ m})^3$$

$$V_E = 1.07 \times 10^{21} \text{ m}^3$$

The mass of the Earth is then:

$$M_E = (5,500\,\frac{\text{kg}}{\text{m}^3})(1.07 \times 10^{21} \text{ m}^3)$$

$$M_E = 5.90 \times 10^{24} \text{ kg}$$

If you squish this down to the density of a white dwarf, you get a volume:

$$M = V\,d$$

$$V = \frac{M}{d} = \frac{5.90 \times 10^{24} \text{ kg}}{10^9 \text{ kg m}^{-3}}$$

$$V = 5.9 \times 10^{15} \text{ m}^3 = \frac{4}{3}\pi R^3$$

Solve this for the radius:

$$R = \left(\frac{3}{4\pi}(5.9 \times 10^{15}\,3)\right)^{1/3}$$

$$R = 10^5 \text{ m} = 100 \text{ km}$$

The diameter will be twice that.

There is a more direct way to do this that involves less math. For both the real Earth and the Earth-mass white dwarf, we have the mass-volume-density equation:

$$M_E = V_E\,d_E = \frac{4}{3}\pi R_E{}^3\,d_E$$

$$M_{wd} = V_{wd}\,d_{wd} = \frac{4}{3}\pi R_{wd}{}^3\,d_{wd}$$

We have, however, $M_E = M_{wd}$, since it is the earth we're squinching down to the density of a white dwarf. Since it's size we care about, solve each of these equations for Radius:

$$R_E = \left[\frac{3}{4\pi}\frac{M_E}{d_E}\right]^{1/3} = R_E$$

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$$R_{wd} = \left[\frac{3}{4\pi}\frac{M_{wd}}{d_{wd}}\right]^{1/3} = R_{wd}$$

Now divide the two equations:

$$\frac{R_{wd}}{R_E} = \left[\left(\frac{M_{wd}}{M_E}\right)\left(\frac{d_E}{d_{wd}}\right)\right]^{1/3}$$

Everything else divides out, and indeed since $M_{wd} = M_E$ we're left with:

$$\frac{R_{wd}}{R_E} = \left(\frac{d_E}{d_{wd}}\right)^{1/3}$$

As diameter is twice the radius, we can get:

$$\text{diam}_{wd} = \left(\frac{d_E}{d_{wd}}\right)^{1/3}\text{diam}_E$$

$$\text{diam}_{wd} = \left(\frac{5500}{10^9}\right)^{1/3}(12,700\,\text{km})$$

$$\boxed{\text{diam}_{wd} = 200\,\text{km}}$$

(to one sig fig, which is all we've got).

5. Chapter 16, Question 11: *In 1841, the 150 $M_\odot$ star, Eta Carinae, was losing mass at the rate of 0.1 $M_\odot$ per year. Let's put that into perspective.*

   (a) *The mass of the Sun is 2×10^{30} kg. How much mass (in kg) did Eta Carinae lose each minute?*

   (b) *The mass of the Moon is 7.35×10^{22} kg. How does Eta Carinae's mass loss per minute compare with the mass of the Moon?*

   (a)
   $$\left(\frac{0.1\,M_\odot}{\text{yr}}\right)\left(\frac{2\times10^{30}\,\text{kg}}{M_\odot}\right)\left(\frac{1\,\text{yr}}{5.26\times10^5\,\text{min}}\right)$$

   $$= 3.80\times10^23\,\frac{\text{kg}}{\text{min}}$$

   Eta Carina was losing $\boxed{4\times10^{23}\,\text{kg each minute}}$.

   (b)
   $$\left(3.80\times10^{23}\,\text{kg}\right)\left(\frac{M_{\text{Moon}}}{7.35\times10^{22}\,\text{kg}}\right)$$

   $$= 5.17\,M_{\text{Moon}}$$

   Eta Carina was losing $\boxed{5\text{ times the mass of the Moon each minute}}$.

   Note that we only have one significant figure here, even though the Moon mass was given to three; the *other* number is only good to one figure. Notice also that I kept an extra couple of digits for intermediate calculations to avoid errors due to repeated rounding.