Astronomy 102, Fall 2003

Homework Set 9 Solutions and Commentary

1. Chapter 16, Question 13: *In our Galaxy, there are about 50,000 stars of average mass ($0.5\,M_\odot$) for every main sequence star of mass $20\,M_\odot$. But stars with $20\,M_\odot$ are about 10^4 times more luminous than the Sun, and $0.5\,M_\odot$ stars are only 0.08 times as luminous as the Sun.*

(a) How much more luminous is a single massive star than the total luminosity of the 50,000 less massive stars?

(b) How much mass is in the lower-mass stars compared to the single high-mass star?

(c) What does this tell you about which stars contain the most mass in the Galaxy and which stars produce the most light?

(a)

$$\frac{10^4\,L_\odot}{(50,000)(0.08\,L_\odot)} = 2.5$$

The single $20\,M_\odot$ star is $\boxed{\text{3 times as luminous}}$ as all 50,000 average stars put together! (Notice that I did *not* need to convert this to Watts in order to do the problem. Many of you did, which is fine, but was extra, unnecessary work.)

(b)

$$\frac{(50,000)(0.5\,M_\odot)}{20\,M_\odot} = 1250$$

The average stars have $\boxed{\text{1000 times the mass}}$ of the $20\,M_\odot$ star. *NOTE!* This ratio does *not* have units! The units of $M_\odot$ cancelled out in the quotient aboe.

(c) All you can really conclude is: although average stars have much more of the Galaxy's mass than luminous main sequence stars, those luminous main sequence stars put out more light than all those average stars put together.

Note: A few of you made the equation "more mass equals more luminosity". This only works *on the main sequence.* It is true that more massive main sequence stars are more luminous. However, an 0.8 $M_\odot$ red giant is much more luminous than a 1.2 $M_\odot$ main sequence star, even though the latter is more massive.

Note: Don't confuse high-mass main sequence stars with red giants! They are not at all the same thing!

Note: A lot of you concluded far too much, although I did not take points off for this. Some things you should not have been able to conclude:

Most of the light of the galaxy comes from high-mass stars. We haven't considered all types of stars here. Even if we assume that the problem gives accurate result for high mass versus low mass main sequence stars, we haven't considered giant stars. In fact, low mass stars do get a chance to contribute significantly to the luminosity of a galaxy when they are briefly giants, and much more luminous. (How much red giants versus massive main sequence stars contribute depend on the type and age of the stellar population.)

Most of the Galaxy's mass is in low mass stars. We now know from what we've done in class since this homework set was due that in fact most of the Galaxy's mass is in dark matter! All this problem really tells us is that the low mass stars use up more of the mass than the main sequence stars that put out the most light; or, equivalently, the low mass stars have a much higher percentage of the mass of the galaxy, but the high mass stars put out more light. You might also say that most of the *stellar* mass of the Galaxy is in low-mass stars.

2. *Type Ia and Type II Supernovae are each explosions that signify the death of a star and which are briefly as luminous as an entire galaxy. However, they are very different sorts of objects. What is the original source of the energy which powers each type of explosion? (I.e. where did the energy come from that allowed the energy in the explosion to be released?)*

Type Ia: fusion of carbon to heavier elements

Type II: gravitational potential energy released as the core collapses to a much smaller size

Notes: That's all that this problem needed. Many of you who got this also embellished your answer with a page of exposition about what happens in each sort of supernova. That's fine, but if you're on a test, you should conserve your time by answering the question which was asked.... If something helps you elucidate the answer, then it's great, but extraneous information isn't necessarily useful.

However, several simply *didn't answer the question which was asked*! A lot of you answered the question "Please describe a Type Ia and a Type II supernova each in a paragraph or two." A few answered the question "What is the difference between a Type Ia and a Type II supernova?" A few more answered the question "What sort of star or star system do you have to have in order to get each type of supernova?" These answers resulted in a lot of (sometimes) very nice text saying what goes on in each type of supernova, but which never identified where all the energy which is released in each event comes from. I did give a point of partial credit for these nice descriptions, but no more than that since you didn't answer the question which was asked.

Take note of this as a strategy for answering problems. Sometimes, a huge information dump may accidentally spit out the right answer somewhere in the middle of everything else. Make sure, however, that you've thought about what the question is asking, and that you've actually addressed that in your answer.

3. *When you look at a globular cluster through a small telescope with your eye, the individual stars you can see are largely red giants.*

(a) Why is this so?

(b) Why aren't you seeing any high-luminosity massive main sequence stars?

(c) Is what you are seeing representative of the population of stars in the globular cluster? Why or why not?

Most of this problem could be answered by thinking about and understanding the H-R diagram of a globular cluster, e.g. such as is found in Figure 16.17 in your text (and perhaps comparing that with the "cluster snapshot" diagrams on the previous page).

(a) Because the red giant stars are the most luminous stars in globular clusters. Just as the stars we see most easily in the night sky with our naked eye are the brightest ones, when we look at a globular cluster the stars we will see most easily (and which will be readily apparent to a small telescope) will be the brightest ones. All the stars in a globular cluster are very close to being at the same distance from us, so the most luminous ones are the ones that will appear brightest.

Note: A number of you said that red giants were the brighest because they are very massive and very luminous. They are not very massive! They are what stars become after they are done with the main sequence. When our Sun becomes a red giant, it will be much more luminous than it is now, but it will certainly be no more massive.

(b) Because they're long dead. High-luminosity main sequence stars only live millions, or tens of millions of years, whereas typical globular clusters are more than 10 billion years old. You can see this in the H-R diagram in Figure 16.17 in your text; there are no stars up in the massive, blue end of the main sequence. **Note:** A number of you said that the massive stars had all become red giants. While that is true, that was only for a brief period of time. (Technically, the most massive stars became supergiants, but we aren't worrying about that distinction in this class.) The most massive stars went supernova long ago; the ones that were several solar masses went through their red giant phase and became white dwarves long ago.

(c) No. Refer back to problem 1. Most of the stars in a globular cluster are low-mass main sequence stars who are burning their nuclear fuel slowly enough that they haven't yet had time to leave the main sequence. In contrast, most of the light is coming from the most luminous stars (which in this case are the stars with the largest radius; while they are more massive than stars who haven't yet left the main sequence (since all the stars formed at once), they are only a little bit more massive, in contrast to the difference in Problem 1).

4. Chapter 15, Question 9: *In Latin, Nova means new. Novae, as we now know, are not "new" stars. Explain how novae might have gotten their name.*

On a good, dark night (from a site well away from city lights), you can see several thousand stars with your naked eye. This is only a tiny fraction of the stars in the galaxy; we only see the brightest stars, be it because they are luminous enough or close enough. In a nova, the star temporarily becomes a whole heck of a lot more luminous than it normally is. Indeed, normally, the luminosity of the white dwarf is much lower than the luminosity of the red giant from which it is pulling mass. However, during the nova event, the white dwarf briefly outshines the red giant. Even if the red giant is not bright enough for us to see with our naked eye, the nova may well be. As such, looking out at the sky, we may see a star where we hadn't been able to see one before. This would appear to us as a new star, especially if we didn't have the benefit of a telescope to realize just how many stars are out there.

5. Chapter 15, Question 13: *A white dwarf has a density of approximately 10^9 kg/m^3. Earth has an averate density of 5,500 kg/m^3 and a diameter of 12,700 km. If Earth were compressed to the same density as a white dwarf, how large would it be?*

The volumne of a sphere is:

$$V = \frac{4}{3}\pi R^3$$

Density is:

$$d = \frac{M}{V}$$

What we care about is size, so for the time being let's worry about Volume. (We'll turn that into radius later.) We know density, and we know that the mass doesn't change (it's the whole Earth being compressed into a white dwarf) (which would be really sad, by the way). Solve the density equation:

$$V = \frac{M}{d}$$

Now divide the two for E (Earth) and wd (Earth as a white dwarf):

$$\frac{V_{wd}}{V_E} = \frac{M_{wd}/d_{wd}}{M_E/d_E}$$

$$\frac{V_{wd}}{V_E} = \left(\frac{M_{wd}}{M_E}\right)\left(\frac{d_E}{d_{wd}}\right)$$

We just said that the mass stays the same, so $M_{wd}/M_E = 1$:

$$\frac{V_{wd}}{V_E} = \frac{d_E}{d_{wd}}$$

Finally, we want radius, not volume:

$$\frac{4/3\ \pi\ {R_{wd}}^3}{4/3\ \pi\ {R_E}^3} = \frac{d_E}{d_{wd}}$$

$$\frac{R_{wd}}{R_E} = \left(\frac{d_E}{d_{wd}}\right)^{1/3}$$

$$R_{wd} = R_E\left(\frac{d_E}{d_{wd}}\right)^{1/3}$$

$$R_{wd} = (12,700\,\mathrm{km})\left(\frac{5,500}{10^9}\right)^{1/3}$$

$$\boxed{R_{wd} = 200\,\mathrm{km}}$$

(To one significant figure.)

6. Chapter 16, Question 12: *If the Crab Nebula has been expanding at an average velocity of 3,000 km/s since A.D. 1054, what was its average radius in the year 2002? (There are approximately* 3×10^7 *seconds in a year.)*

It started from (effectively) size zero. (Really the size of the star that went supernova, but we will see that that is much less than the size of the nebula now.)

$$d = v\,t$$

$$d = \left(3,300\,\frac{\mathrm{km}}{\mathrm{s}}\right)(948\,\mathrm{years})\left(\frac{3.16 \times 10^7\,\mathrm{s}}{\mathrm{year}}\right)$$

$$d = 9.9 \times 10^{13}\,\mathrm{km} = 10\,\mathrm{light-years}$$