Astronomy 102: Stars and Galaxies

Fall, 2003

Sample Review Examination 1 Solutions

- **1.** (d)
- **2.** (b)
- **3.** (c). Anywhere on the Earth (except exactly at the poles), a star on the equator is always up for half of the day. Your horizon exactly bisects the celestial equator, with half of the equator above the horizon, and half below.
- **4.** (a)
- **5.** (c)
- **6.** (e)
- **7.** (a)
- 8. (b). If you have no axial tilt, but your orbit is elliptical enough that the Sun is significantly closer in the summer than the winter, then the summer will happen at that part of your orbit where you're closer to the Sun. As we know from Kepler's second law, a planet in an elliptical orbit moves faster when it is closer to the Sun, so it gets through that part of the orbit than it does the part where the planet is distant from the Sun.
- 9. (a) The equator.
 - (b) Half. You can see half of the celestial sphere at any time from the North Pole (everything at positive declination).
- 10. (a) The sun would rise about once a month. (That's all you would have needed to write on the test in response to this question.) On the Earth, the time between one sunrise and the next is one day, or approximately the rotation period of the Earth. The Moon rotates at the same rate it orbits the Earth, so the rotation period of the Moon is approximately once a month.
 - (b) The Earth would never set! (Again, that's all you would have to write in response to this question on the test.) The same face of the moon is always pointing towards Earth. If you're on that face, and Earth is high in your sky, as the Moon goes around the Earth it rotates at just the right rate so that that face is always pointing at the Earth, and the Earth will stay high in your sky.

p

11. (a)
$$d = \frac{1}{2}$$

with d in pc and p in arcseconds. Thus, we have:

$$d = \frac{1}{0.379} \text{ pc} = 2.64 \text{ pc}$$

(b) To use the parallax equation, we first need to convert this distance to parsecs:

$$430 \operatorname{lyr}\left(\frac{1 \operatorname{pc}}{3.26 \operatorname{lyr}}\right) = 132 \operatorname{pc}$$

Now we can solve the parallax equation for p:

$$p = \frac{1}{d}$$

where d is in parsecs and p is in arcseconds. Thus:

$$p = \frac{1}{132}'' = 0.0076''$$

12. (a)

$$P^2 = A^3$$

where P is the period (time it takes to go around the Sun in years) and A is the average distance from the Sun. Solving this for P involves raising both sides to the 1/2 power:

$$P = A^{3/2}$$

which, remembering that something to the 1/2 power is the same as the square root of that something, can be calculated as:

$$P = \sqrt{A \times A \times A}$$

For Quaoar, we have:

$$P = \sqrt{39 \times 39 \times 39}$$
 years = 280 years

(282.0 rounded to two significant figures).

(b) It is impossible to state from the information provided. We have no information about the eccentricity of Quaoar's orbit. While we know its *average* distance from the Sun is greater than Neptune's or Pluto's, we don't know how close it gets.

$$F = \frac{G M_S M_Q}{r^2}$$

We have to make sure to use the right variables! We can find G, M_S , and M_Q on the front of the test. We need r, which is the distance between the Sun and Quaoar (*not* the radius of the Sun, which was put on the front of the test as a red herring) in meters. We have it in AU, so we need to convert:

$$(43 \text{ AU}) \left(\frac{1.496 \times 10^{11} \text{ m}}{1 \text{ AU}}\right) = 6.433 \times 10^{12} \text{ m}$$

(I'm keeping extra digits for numbers in intermediate calculations beyond the total number of significant figures I'll report in the answer.) Now that we have this, we can plug into the gravitational force equation:

$$F = \frac{(6.67 \times 10^{-11} \,\mathrm{m^3 \ kg^{-1} \ s^{-2}})(2.00 \times 10^{30} \,\mathrm{kg})(2.5 \times 10^{21} \,\mathrm{kg})}{(6.433 \times 10^{12} \,\mathrm{m})^2}$$
$$F = 8.1 \times 10^{15} \,\mathrm{N}$$

(b) Knowing Quaoar's mass and the force from the previous problem, we can calculate Quaoar's acceleration:

$$a = \frac{F}{m} = \frac{8.059 \times 10^{15} \,\mathrm{N}}{2.5 \times 10^{21} \,\mathrm{kg}} = 3.223 \times 10^{-6} \,\frac{\mathrm{m}}{\mathrm{s}^2}$$

If Quaoar is kept moving in a circle of radius 43 AU, then this must be the acceleration needed given its velocity. This, we can work backwards to find the velocity by solving:

$$a = \frac{v^2}{r}$$
$$v = \sqrt{ar}$$

We calculated r in meters in the previous part, so we can plug in:

$$v = \sqrt{(3.223 \times 10^{-6} \frac{\text{m}}{\text{s}^2})(6.433 \times 10^{12} \text{m})}$$
$$v = 4600 \frac{\text{m}}{\text{s}} = 4.6 \frac{\text{km}}{\text{s}}$$

(c) We need to figure out Earth's orbital speed around the Sun. We can do this from F = ma knowing that $a = m v^2/r$ and that F comes from gravity:

$$F = \frac{G M_S M_E}{r^2} = ma = M_E \frac{v^2}{r}$$

Solve this for v, which is what we want:

$$v = \sqrt{\frac{G M_S}{r}}$$

Now we can plug in, remembering that r is the distance from the Sun to the Earth, or 1 AU:

$$v = \sqrt{\frac{(6.67 \times 10^{-11} \,\mathrm{m^3 \ kg^{-1} \ s^{-2}})(2.00 \times 10^{30} \,\mathrm{kg})}{1.496 \times 10^{11} \,\mathrm{m}}}$$
$$v = 29,900 \,\mathrm{ms} = 29.9 \frac{\mathrm{km}}{\mathrm{s}}$$

How many times faster is Earth's orbital speed than Quaoar?

$$\frac{29.9 \,\mathrm{km \ s^{-1}}}{4.55 \,\mathrm{km \ s^{-1}}} = 6.6 \text{ times faster}$$



The asteroid takes *longer* to go around the Sun than does Mars (which you could figure out from $P^2 = A^3$ and the average distance of Mars from the Sun as given on the front of the test; you'd get 1.88 years). Therefore, the asteroid must have a larger *average* distance from the Sun than Mars. However, right now it's closer to the Sun than even Earth...this means for it to have an average distance which is greater than Mars' average distance, it will need to be in an elliptical orbit which extends well beyond the orbit of Mars.