Astronomy 102: Stars and Galaxies

Fall, 2003

Sample Review Examination 3 Solutions

1. (a). Yeah, there were two c's. This means the first one.

2. (a), (f)

3. (b). This is just like the "color" ratio we talked about in class.

4. (a), (c), (d), and just perhaps (g)

5. (e)

6. (c)

7. (b), (d), (f) if it had been written properly, (g)

8. (c)

9. (a) The orbital velocities of stars and gas clouds farther away from the center of the galaxy are *too high* compared to what we would expect from all the matter we can see in the gravity. In order for those stars to stay in circular orbits, and not fly out to either larger orbits or even into intergalactic space, there must be *more* gravity than we can account for from the luminous matter.

(b) Most of the stars are in the disk of the galaxy, and those stars are concentrated more towards the center (they get lower and lower denisty as you get farther from the center of the galaxy). Dark matter is in an big, elliptical or spherical distribution around the whole galaxy, and extends out to much greater distances than does the disk of stars. It is still more dense towards the center of the galaxy, but is in a bigger and smoother distribution than the stars.

10.

$$L_B = 4\,\pi\,{R_B}^2\,\sigma\,{T_B}^4$$

$$\frac{L_B}{4\,\pi\,\sigma\,{T_B}^4} = {R_B}^2$$

Similarly, for the Sun we can get:

$$\frac{L_\odot}{4\,\pi\,\sigma\,{T_\odot}^4} = {R_\odot}^2$$

What we want is $R_B/R_\odot$, so divide the two:

$$\left(\frac{R_B}{R_\odot}\right)^2 = \frac{\frac{L_B}{4\pi\ \sigma {T_B}^4}}{\frac{L_\odot}{4\pi\ \sigma {T_\odot}^4}}$$

$$\left(\frac{R_B}{R_\odot}\right)^2 = \frac{(L_B)(4\pi\ \sigma {T_\odot}^4)}{(L_\odot)(4\pi\ \sigma {T_B}^4)}$$

Divide out the stuff that appears on the top and on the bottom to get:

$$\left(\frac{R_B}{R_\odot}\right)^2 = \left(\frac{L_B}{L_\odot}\right)\left(\frac{T_\odot}{T_B}\right)^4$$

Take a square root of both sides (remebering that something to the 1/2 power is the same as the square root of that something):

$$\frac{R_B}{R_\odot} = \left(\frac{L_B}{L_\odot}\right)^{\frac{1}{2}}\left(\frac{T_\odot}{T_B}\right)^2$$

Plug in the values we know:

$$\frac{R_B}{R_\odot} = (50,000)^{\frac{1}{2}}\left(\frac{1}{5}\right)^2$$

$$\boxed{\frac{R_B}{R_\odot} = 9}$$

to one significant figure.

11. (a)

$$\lambda_{\text{peak}} = \frac{2900K}{T}\ \mu\text{m}$$

$$\lambda_{\text{peak}} = \frac{2900K}{20,000K}\ \mu\text{m}$$

$$\boxed{\lambda_{\text{peak}} = 0.14\,\mu\text{m} = 1400\text{Å}}$$

(b) The Sun is way the heck bigger.

(c)

$$L = 4\pi R^2\ \sigma T^4$$

$$L = 4\pi\,(6.96\times 10^8\,\text{m})^2(5.67\times 10^{-8}\,\frac{\text{W}}{\text{m}^2\,\text{K}^4})(20,000\,\text{K})^4$$

$$L = (5.52\times 10^{28}\,\text{W})\left(\frac{L_\odot}{3.85\times 10^{26}\,W}\right) = \boxed{140\,\text{L}_\odot}$$

(d) We need $F_* = F_\odot$, so:

$$\begin{aligned}
\frac{L_*}{4\,\pi\,{d_*}^2} &= \frac{L_\odot}{4\,\pi\,{d_\odot}^2} \\
L_*\,4\,\pi\,{d_\odot}^2 &= L_\odot\,4\,\pi\,{d_*}^2 \\
d_* &= \sqrt{\frac{L_*}{L_\odot}}\,d_\odot \\
d_* &= \sqrt{140}\,(1\,\mathrm{AU}) \\
\Aboxed{d_* &= 12\,\mathrm{AU}}
\end{aligned}$$

(e)

$$A = \frac{d}{D} = \frac{2\,R_\odot}{12\,\mathrm{AU}}$$

The small angle formula works for the angle in radians when you divide two numbers of consistent units, so we have to do some conversions. (The clever person will see already that the star will appear with 1/12 the angular diameter of the Sun.)

$$A == \left(\frac{2\,R_\odot}{12\,\mathrm{AU}}\right)\left(\frac{1\,\mathrm{AU}}{1.5\times10^{11}\,\mathrm{m}}\right)\left(\frac{6.96\times10^{8}\,\mathrm{m}}{1\,R_\odot}\right)$$

$$A = 7.7\times10^{-4}\,\mathrm{rad}\left(\frac{180^\circ}{3.1416\,\mathrm{rad}}\right) = \boxed{0.044^\circ}$$

This is, in fact, about 1/12 the angular diameter of the Sun.

(f)

$$\begin{aligned}
L &= 4\,\pi\,R^2\,\sigma\,T^4 \\
R &= \sqrt{\frac{L}{4\,\pi\,\sigma\,T^4}} \\
R &= \sqrt{\frac{3.85\times10^{26}\,\mathrm{W}}{4\,\pi\,(5.67\times10^{-8}\,\frac{\mathrm{W}}{\mathrm{m^2\,K^4}})\,(20,000\,\mathrm{K})^4}} \\
R &= (5.8\times10^{7}\,\mathrm{m})\left(\frac{1\,R_\odot}{6.96\times10^{8}\,\mathrm{m}}\right) \\
\Aboxed{R &= 0.08\,R_\odot}
\end{aligned}$$

12.

(a) This tells us one of two things. First, it is possible that the Universe is *not* infinte, and is surrounded by "empty space"— that is, there is a point beyond which you see no more stars, and we an see to that point. (Our *Galaxy* is like this, and indeed all of the stars we see with our naked eye are in our Galaxy, but there are other galaxies in the Universe beyond our own, and our Universe is not like this.) The second possibility is that light takes a finite amount of time to travel, and the Universe hasn't always been here— it isn't infinite in *time*, at least looking backwards.

(b) The first is, of course, the fact that the Big Bang theory tells us that the Universe is only some 13 or 14 billion years old. The second is the redshift that results from an expanding Universe. Not only can light only reach us from a certain distance away (i.e. the distance that corresponds to how far light can have travelled since the beginning of the Universe), but as that light travels and the Universe expands, photons are shifted to *lower* energy, thereby reducing the energy density we'll see on the sky. There is a third reason, and that is that the Big Bang theory tells us the Universe is evolving. As you look far away, you're looking back in time, and eventually you're looking back to a time before the first stars had formed.