Astronomy 102, Fall 2004

Review Final Solutions

1. A high-mass star lives no more than about 80 million years (the lifetime of an 8 $M_\odot$ star); more massive ones live less time. A star with this lifetim ewill make it about 1/3 of the way around the galaxy before it pops; it never even complets one orbit. Stars that only live 10 million years only make it 1/25 of the way around.

The high mass stars that are forming right now will go supernovae long before the Sun gets out of the main sequence stage of stellar evolution. The Sun's got another 5 billion years on its clock, which is much longer than the lifetime of even an 8 $M_\odot$ star.

2. **(a)** Planet A. It has the shorter period. Given that the two stars have the same mass, it must be closer.

(b)

$$
\begin{aligned}
{P_A}^2 &= \left(\frac{4\pi^2}{GM}\right) {R_A}^3 \\
{P_B}^2 &= \left(\frac{4\pi^2}{GM}\right) {R_B}^3
\end{aligned}
$$

M is the same in both cases, since the star for each planet has the same mass. Divide these two equations to get:

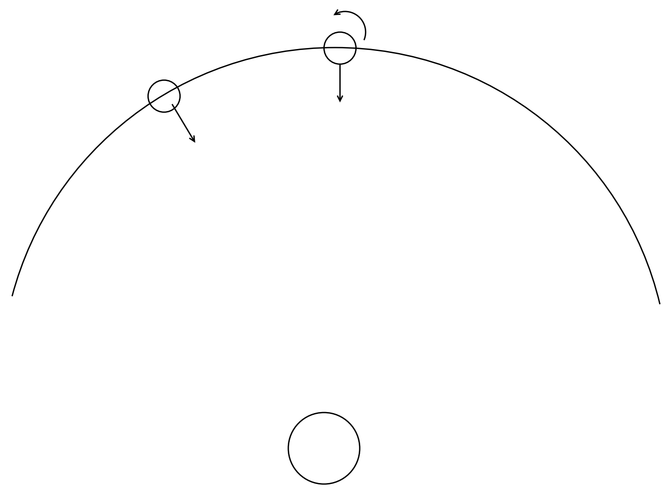
$$
\begin{aligned}
\left(\frac{P_A}{P_B}\right)^2 &= \left(\frac{R_A}{R_B}\right)^3 \\
\frac{R_A}{R_B} &= \left(\frac{P_A}{P_B}\right)^{\frac{2}{3}} \\
\frac{R_A}{R_B} &= \left(\frac{1}{2}\right)^{\frac{2}{3}} \\
\boxed{\frac{R_A}{R_B} = 0.63}
\end{aligned}
$$

(c) Planet B must be more massive. The wobbling of the stars is due to the gravity of the planets. Star B shows more wobbling, so it must feel stronger gravity from its planet than Star A does. However, Planet A is closer to its star, so the only way for Planet B to be exerting more gravity on its star is if it is quite a bit more massive than Planet A.

3. **(a)** Low in the Western sky.

(b) The tilt of the Earth's axis relative to the plane of it's orbit. The axis would need to have no tilt, i.e. be perpendicular to the plane of the orbit.

(c) The length of the full day will need to be a bit *longer* in the winter. For the Sun to go from overhead one day to overhead the next day, the Earth has to make more than one complete rotation. It has to rotate a little extra bit in order to account for the fact that it's moved a little bit around the Sun:



In the Winter, when the Earth is closer to the Sun, it is moving a bit faster (Kepler's Second Law). This means that it covers more of its orbit in one day than it does during the Summer, and therefore it will have to have a little bit more of this extra rotation in order to get the Sun overhead again.

When we usually say "days are shorter in winter," we're talking about the number of hours of daylight, not the number of hours it takes to go from the Sun being as high in the sky as it gets one day to the Sun being as high in the sky as it gets the next day.

(d) Almost everywhere; everywhere except for right at the equator.

4 (a) The two stars are orbiting each other, which means they're a binary star system. Almost certainly they formed together, so they are the same age. In any event, they are the same distance from each other, so the ratio of brightnesses will be the same as the ratio of luminosities.

$$\frac{L_W}{L_R} = \frac{4\pi\, {R_W}^2\, \sigma\, {T_W}^4}{4\pi\, {R_R}^2\, \sigma\, {T_R}^4}$$

$$\frac{L_W}{L_R} = \left(\frac{R_W}{R_R}\right)^2 \left(\frac{T_W}{T_R}\right)^4$$

$$\frac{L_W}{L_R} = \left(\frac{6,378\,\mathrm{km}}{6.96\times 10^7\,\mathrm{km}}\right)^2 \left(\frac{6,000\,\mathrm{K}}{3,000\,\mathrm{K}}\right)^4$$

$$\boxed{\frac{B_W}{B_R} = \frac{L_W}{L_R} = 1.3\times 10^{-7}}$$

The white dwarf, despite being hotter, is much, much dimmer.

(b) I meant *same mass* rather than *same radius*, but I'll answer both.

First of all, it's not possible that they have the same radius, because, er, we're told that they're not the same.

As for the same mass: a white dwarf is a *later* stage of stellar evolution than is a red giant. If the two stars formed at the same time, that means that the star that is now a white dwarf must have *started* higher mass than the other star, so that it would now be farther along in its evolution. However, stars lose mass as red giants, and as they become white dwarves they throw off a planetary nebula and lose more mass. So, now, yes, it is possible that the two stars have the same mass. The white dwarf would have started with higher mass, but may have lost enough mass to get down to the mass of the red giant. (Indeed, it's possible that now the white dwarf has a lower mass, even though when it was a main sequence star it would have had to have been higher mass.)

(c) We have the luminosity ratio of the two stars from (a). We want the two stars to have the same brightness:

$$\begin{aligned} B_{W2} &= B_R \\ \frac{L_W}{4\pi\, {d_{W2}}^2} &= \frac{L_R}{4\pi\, {d_R}^2} \\ d_{W2} &= \sqrt{\left(\frac{L_W}{L_R}\right)}\, d_R \\ d_{W2} &= \sqrt{1.3\times10^{-7}}\; 500\,\mathrm{lyr} \\ &\boxed{0.18\ \mathrm{light-years}} \end{aligned}$$

That's damn close.

5. **(a)** It hasn't been through as many generations of star formation as most of the gas in the disk of the Milky Way.

(b) Maybe not; it's very difficult to say anything. *If* the fractional star formation rate now is similar to that in the Milky Way, then the other galaxy will be bluer because the fraction of stars that have recently been formed is higher in that galaxy than in the Milky Way (where there are lots of long-lived red stars left over from previous generations of star formation).

(c) Higher in the other galaxy. Each time you go through a generation of star formation, some of the mass of gas is "lost" to long-lived low mass stars (as well as stellar remnants like white dwarves and neutron stars). This means that not all of the gas gets returned to make future generations. The Milky Way has been through more generations, so a greater fraction of its gas has been "lost" to long-lived low-mass stars.

6. **(a)** A Type II supernova. This $12\,M_\odot$ star is gonna pop. (In fact, it has already. High-mass stars live less than 80 million years, and the red giant stage is $\sim 10\%$ of a star's life, so this star is definitely less than 65 million years from the end.)

(b) Probably a neutron star, left over from the supernova remnant.

(c)

$$\begin{aligned} v &= H_0\, d \\ v &= \left(72\,\frac{km/s}{Mpc}\right)(65Mlyr)\left(\frac{1\,\mathrm{pc}}{3.26\,\mathrm{lyr}}\right) \\ &\boxed{v = 1400\,\mathrm{km/s}} \end{aligned}$$

7. Gas and stars that are far away from the center of the galaxy are moving *faster* than we would expect them to given the amount of gravity holding them to the galaxy from all the stars and gas we can see. This means that the galaxy ought to be flying apart... but it's not. Thus, there must be more mass holding it together than we can see; that mass we haven't been able to see is what we call dark matter.

8. The rotating star ought to have a lower surface temperature. Each star has the same amount of gravity at its surface; same mass, same radius. To keep the star in equilibrium, the atoms at the surface of the star need the same amount of motion. For stars that aren't rotating, all of that motion comes from pressure. For a star that is rotating, some of that motion comes from the bulk motion of the star rotating, so it needs less pressure in order to hold it up. If the pressure is less, either it's less dense (unlikely for two stars of the same size and mass), or the atoms aren't moving around as fast. If they aren't moving as fast, that's a lower temperature.