Astronomy 102, Fall 2004

Homework Set 3 Solutions

1. *The 6563Å line of Hydrogen from the Andromeda galaxy is observed at 6556Å.*

(a) *Is the Andromeda galaxy moving towards us or away from us?*

(b) *How fast (at what speed) is the Andromeda galaxy moving along the line of sight?*

(c) *If the Milky Way and Andromeda galaxies are 2.6 million light-years apart, and you assume that there is no tangential velocity (velocity perpendicular to the line of sight), how long will it be before the two galaxies either collide, or until the distance between them doubles? (Indicate which will happen.)*

(a) It is moving $\boxed{\text{towards}}$ us. We observe the line at a lower wavelength, which is bluer light. A blueshift happens when something is approaching you.

(b)

$$\frac{v}{c} = \frac{\lambda_{\rm obs} - \lambda_0}{\lambda_0}$$

$$v = \left(\frac{6556\text{Å} - 6563\text{Å}}{6563\text{Å}}\right)(3.00 \times 10^8 \text{ m/s})$$

$$v = (-0.001)(3 \times 10^5 \text{ km/s})$$

$$\boxed{v = 300 \text{ km/s towards us}}$$

(c) The distance between the two galaxies is about 2.6 million light years. You can lookup that 1 lyr$=9.5 \times 10^{12}$ km. Then:

$$d = vt$$

$$t = \frac{d}{v}$$

$$t = \frac{(2.6 \times 10^6\,\text{lyr})(9.5 \times 10^{12}\,\text{km}\,\text{lyr}^{-1})}{300\text{ km}\,\text{s}^{-1}}$$

$$t = (8.23 \times 10^{16}\,\text{s})\left(\frac{1\,\text{yr}}{3.16 \times 10^7\,\text{s}}\right)$$

$$t = 2.6 \times 10^9\, yr$$

So the two galaxies will collide, if they're coming right towards each other, in $\boxed{\text{2.6 billion years}}$.

2. Pasachoff & Filippenko, 2.9: *Describe how the same atoms can sometimes cause emission lines and at other times cause absorption lines.*

An emission line comes when an atom transitions from a more-excited energy state to a less-excited energy state (perhaps the ground state). When it does this, it emits a photon whose energy is the same as the difference in the energies of the two states. If you have atoms who are (somehow) in excited energy states, they can emit photons.

The same atoms can absorb light if you shine light on them that includes some photons at exactly the right wavelengths so that the photon energies correspond to the difference of energies of the atom's states. In this case, the atom will take a photon and use its energy to move from a less excited state to a more excited state.

3. *The luminosity of the Sun (i.e. the amount of energy it puts out each second) is* 3.9×10^{26} *W. It radiates this energy approximately as a blackbody.*

(a) If the radius of the Sun were to double while keeping its surface temperature the same, what would be its luminosity? How would the color of the Sun change?

(b) If the temperature of the Sun were to double while keeping its size the same, what would be its luminosity? How would the color of the Sun change?

(a) If the radius were to double, but the temperature stays the same, the color will be unaffected. The color of something radiating as a blackbody depends only on the temperature. Hotter is bluer, cooler is redder, whether we're talking a stovetop burner or a supergiant star.

As for the luminosity, remember that the total energy output is equal to the surface area times σT^4. The surface area of a sphere is $4\pi R^2$, so if R doubles, the surface area (and therefore) the luminosity goes up by a factor of 4. $4 \times (3.9 \times 10^{26}\,\mathrm{W})$ is $\boxed{1.6 \times 10^{27}\,\mathrm{W}}$.

b If the temerature were to double, the color would get *bluer*. The luminosity would go up by a factor of $2^4 = 16$, since luminosity depends on T^4 when the emitting area doesn't change. This gives a luminosity of $16 \times (3.9 \times 10^{26}\,\mathrm{W})$, or $\boxed{6.2 \times 10^{27}\,\mathrm{W}}$.

4. When an item moves away from you, its light is redshifted. This means that each individual photon reaching you from the object has *less* energy than it did when it was emitted by the moving object. Since you get less energy from each photon, you're going to get less energy per second overall.

Additionally, the rate at which you receive those photons decrease. If the emitting object is moving away from you, each photon is going to have a bit farther to travel than the one emitted just before it. This means that it will take longer for subsequent photons to reach you, stretching out the time between them. So, not only does each photon that reaches you have less energy, the rate at which photons reach you goes down. Both of these contribute to reducing the energy you detect from an object moving away from you.

The opposite would hold for an object moving towards you. (If yout take a first year graduate astrophysics class, you will actually calculate this factor as a function of the angle of the object moving; this effect is called "relativistic beaming".)

5. *Atomic emission "lines" as described in class always happen at exactly one wavelength: the wavelength where photons have the same energy as a specific atomic transition. In fact, atronomers usually observe "broadened" lines from astrophysical sources, where a line at a given wavelength is spread into nearby wavelengths. One very common mechanism for this broadening is called "Doppler broadening": if a gas cloud is turbulent, with atoms moving about randomly, some of the atoms in the cloud will be moving towards you, some away from you. There will be a blueshift or a redshift of the light emitted by atmos moving towards or away from you, and as the light from all of the atoms is added together you get a smeared out (or broadened) line. Consider the 6563? line of Hydrogen. Suppose from a nebula it is observed to be spread from 6560-6566?. What range of speeds towards/away from you would you conclude the atoms in this nebula are moving, if you interpret this broadening as a doppler broadening? (Give your answer in km/s.)*

The most negative velocity you will get is when the gas is coming towards you and is therefore blueshifted to the lowest observed wavelength:

$$\begin{aligned} \frac{v_{\min}}{c} &= \frac{6560\,\text{Å} - 6563\,\text{Å}}{6563\,\text{Å}} \\ v_{\min} &= (-0.000457)(3.00 \times 10^8\,\frac{\mathrm{m}}{\mathrm{s}} \\ v_{\min} &= -140,000\,\frac{\mathrm{m}}{\mathrm{s}} \\ v_{\min} &= -140\,\mathrm{km/s} \end{aligned}$$

Similarly, the highest velocity you will get is when the gas is receeding and therefore redshifted to the lowest observed wavelength:

$$\frac{v_{\min}}{c} = \frac{6566\,\text{Å} - 6563\,\text{Å}}{6563\,\text{Å}}$$

$$v_{\rm min} = (+0.000457)(3.00 \times 10^8\,\frac{\rm m}{\rm s})$$
$$v_{\rm min} = 140,000\,\frac{\rm m}{\rm s}$$
$$v_{\rm min} = 140\,{\rm km/s}$$

Thus, the range is $\boxed{\text{-140 to 140 km/s}}$, or equivalently everything between 140 km/s approaching and 140 km/s receeding.

Note that the "everything between" is important! A number of you said "-140 or 140 km/s" or "either -140 or 140 km/s". This is not right! In that case, you would see a split line, with two very narrow lines at 6560Å and 6563Å. To get a *broadened* line, you need a continuous range of velocities.

A note on significant figures: Really the answer to this problem should only have *one* significant figure! I only took off a point if you reported more than three. Why only one, when it looks like the initial numbers are given to four sig figs? Go back and look at the Math Review again. Notice that you are subtracting two numbers, 6560-6563. That subtraction is only significant to the *ones* place, and that's the point at which your number of significant figures gets reduced to one.