Astronomy 102, Fall 2004

Homework Set 4 Solutions

1. *Compare radio photons (assume a wavelength of 1 m), and ultraviolet photons (assume a wavelength of 100 nm, remembering that one nanometer (nm) is* 10^{-9} *m).*

(a) *If you have the same amount of energy in both kinds of photons, will you have the same number of photons, or will there be more of one kind? If so, of which kind will there be more?*

(b) *How many photons of one type does it take to make up the energy of a single photon of the other type? Indicate clearly which type(s) of photon you have only one of.*

(a) If you have the same *energy*, you're going to need many, many more radio photons. Energy is proportional to frequency, but frequency is inversely proportional to wavelength. This means that as the wavelength of a photon goes up, the energy of that photon goes down. Radio photons have a much longer wavelength, and therefore each photon has a much smaller energy than each ultraviolet photon.

(b) The energy in one photon is:

$$E = \frac{h\,c}{\lambda}$$

We have just one UV photon, and need some number N radio photons such that:

$$\begin{aligned} N\,E_R &= E_{UV} \\ N\left(\frac{h\,c}{\lambda_R}\right) &= \left(\frac{h\,c}{\lambda_{UV}}\right) \\ \frac{N}{\lambda_R} &= \frac{1}{\lambda_{UV}} \\ N &= \frac{\lambda_R}{\lambda_{UV}} \end{aligned}$$

This gives a result of $\boxed{N = 10^7}$ radio photons to equal the energy of one UV photon. (Remember that the UV photon has a wavelength of 100 nm, not 1 nm! A lot of you made that mistake.)

2. *Refer to the H-R diagrams on p. 226 and 227 of your text. The star Vega is a star that's about three times as massive as the Sun.*

(a) *What is the spectral type of Vega? Is Vega bluer or redder than the Sun?*

(b) *What is Vega's Luminosity in units Solar Luminosities?*

(c) *What is Vega's temperature in K?*

(d) *Using Figure 11-13 in your text, and what you know about sizes, brightnesses, and temperatures, estimate the radius of Vega in both km and Solar Radii.*

(a) This is just plot reading. From the two plots, we can read off that Vega has a spectral type of A, which makes it bluer than the Sun (type G).

(b) Again reading the plot, we get that Vega is about 50 $L_\odot$. This may surprise some of you, but reading log plots is a little tricky. (I gave full credit for anything between about 40 and 100 $L_\odot$, since I know that without experience reading log plots is hard.)

(c) About 10,000K.

(d) Do it in Solar Radii first; this is easier. We have:

$$\begin{aligned} L_V &= 4\pi\,{R_V}^2\,\sigma\,{T_V}^4 \\ L_\odot &= 4\pi\,{R_\odot}^2\,\sigma\,{T_\odot}^4 \end{aligned}$$

Divide the two equations:

$$\frac{L_V}{L_\odot} = \frac{4\pi\, {R_V}^2\, \sigma\, {T_V}^4}{4\pi\, {R_\odot}^2\, \sigma\, {T_\odot}^4}$$

$$\frac{L_V}{L_\odot} = \left(\frac{R_V}{R_\odot}\right)^2 \left(\frac{T_V}{T_\odot}\right)^4$$

Solve this for the radius ratio:

$$\left(\frac{R_V}{R_\odot}\right)^2 = \left(\frac{L_V}{L_\odot}\right) \left(\frac{T_\odot}{T_V}\right)^4$$

$$\left(\frac{R_V}{R_\odot}\right) = \sqrt{\left(\frac{L_V}{L_\odot}\right)} \left(\frac{T_\odot}{T_V}\right)^2$$

Putting in $T_\odot = 5780K$, and already knowing that $L_V = 50\, L_\odot$ from above, this direcly gives us $\boxed{R = 2.4\, R_\odot}$.

To convert to km, look up 6.96×10^5 km, and multiply it out to get $\boxed{R = 1.6 \times 10^6\, \text{km}}$ for Vega.

3. *Assume that the amount of fuel available to power a star is proportional to its mass. (That is, assume a star is always able to use the same fraction of its mass as fuel for producing energy.) If the Sun is going to shine for 10 billion years, how long will Vega shine?*

A lot of you had a lot of trouble with this.

If Vega has 3 times the mass of the Sun (see previous problem), and the amount of fuel is a constant fraction of the mass, then Vega has three times as much fuel. Many of you concluded that this means that Vega will live three times as long— but that's wrong. Just as the SUV with a larger tank than a compact car won't necessarily be able to go more miles on a single tank, we also have to consider the rate at which the fuel is being used up.

The luminosity of a star is the power output, the energy used up each second. Vega has three times as much fuel, but is using it up 50 times as fast. This means that Vega will only live 3/50 or 0.06 as long as the sun, or for about $\boxed{\text{600 million years}}$.

4. Pasachoff & Filippenko 11.5: *Star A appears to have the same brightness through red and blue filters. Star B appears brighter in the red than in the blue. Star C appears brighter in the blue than in the red. Rank these stars in order of increasing surface temperature*

Star B
Star A
Star C

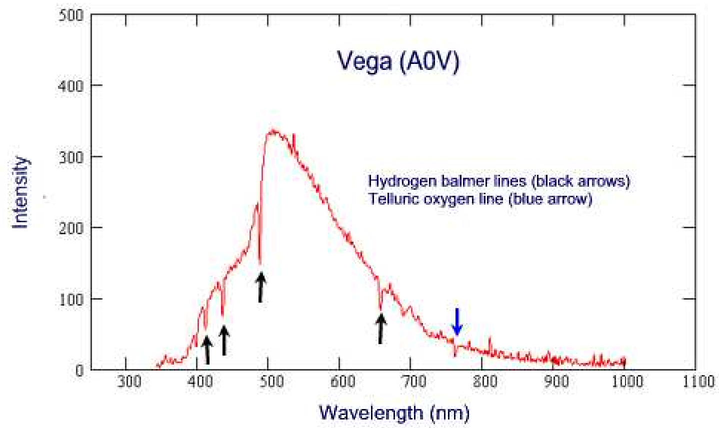
5. Pasachoff & Filippenko 11.6: *What is the difference between continuous radiation and an absorption line? Continuous radiation and an emission line? Graph a spectrum that shows both continuous radiation and absorption lines. Can you draw absorption lines without continuous radiation? Can you darw emission lines without continuous radiation? Explain.*

Continuous radiation is emitted at a wide range of wavelengths, and shows a fairly "smooth" spectrum (no abrupt jumps in a plot of intensity versus wavelength). An absorption line is a *reduction* in the intensity of radiation at or near a very specific wavelength. An emission line is radiation that is all at or very near a specific wavelength.

You *can* have an emission line without anything else there; just have light coming out at a very specific wavelength. Planetary Nebula spectra *almost* look like this (the lines are strong, but there is some continuum), and low pressure vapor lamps (like neon lights) have spectra that look very much like this. (There are several emission lines, but very little continuum.)

You *cannot* have an absorption line without continuous radiation. There has to be something to absorb.... On the spectrum, the absorption line means that the intensity is lower than the continuum on either side. If the continuum is already at zero intensity, however, you can't get any lower.

Here is a spectrum of Vega, which shows both continuum radiation and absorption lines, taken with the Rigel Telescope at Winer Observatory (`http://phobos.physics.uiowa.edu/tech/rigel-tests.html`):



6. Pasachoff & Filippenko 11.19: *Sketch a star's spectrum that continas two spectral lines. Then sketch the spectrum of the same star if the star is moving toward us. Finally, sketch the spectrum if the star is moving away from us.*