Astronomy 102, Fall 2004

Homework Set 5 Solutions

1. ***(a)*** *An astronomer detects two stars which she believes have identical luminosities. Star A has a measured parallax of 0.125". Star B is detected to be 378 times dimmer than Star A. What is the distance to star B in pc?*

(b) *If one star is yellowish, and the other star is make an estimate of Ryellow/Rblue, the ratio of the two stars' radii.*

(a) We know that $d_A = 1/p_A = 8.00\,\text{pc}$. If the two stars have the same luminosities, then the only reason that Star B is dimmer is because it is farther away. (We had better come up with a bigger distance for Star B, therefore, or we'll know we did our math wrong.)

$$B_A = \frac{L_A}{4\pi\, {d_A}^2}$$

$$B_B = \frac{L_B}{4\pi\, {d_B}^2}$$

If you divide these two, you get:

$$\frac{B_A}{B_B} = \left(\frac{L_A}{4\pi\, {d_A}^2}\right)\left(\frac{4\pi\, {d_B}^2}{L_B}\right)$$

Knowing that $L_A = L_B$, we have:

$$378 = \left(\frac{d_B}{d_A}\right)^2$$

$$d_B = \sqrt{378} \times 8.00\,\text{pc}$$

Yielding an answer of $\boxed{d_B = 156\,\text{pc}}$.

(b) To do this one, you need to be able to make an educated guess at the temperatures of the two stars. We know the sun is yellow, so we'll use a Sun-like temperature of 6,000K for the yellowish star. For the bluish star, we know O and B stars are blue, so we'll look at an H-R diagram that lists both spectral types and temperatures along the X-axis, and choose 18,000K for the bluish star.

If the two stars have the same luminosity, then we know:

$$4\pi\, {R_Y}^2\, \sigma\, {T_Y}^4 = 4\pi\, {R_B}^2\, \sigma\, {T_B}^4$$

This is straightforward to rearrange for the ratio in question:

$$\frac{R_Y}{R_B} = \left(\frac{T_B}{T_Y}\right)^2$$

For our choice of temperatures, this gives us a radius ratio of $\boxed{9}$. The yellow star has to be a lot bigger than the hotter blue star to be able to put out the same luminosity.

2. *One of the most famous comets in the solar system is Comet Halley. comet. This comet returns to the inner solar system and is visible every 76 years; it's last apparition was in 1986. The comet, like other periodic comets, is in a highly eccentric elliptical orbit around the Sun.*

(a) *What is the semi-major axis of the orbit of Comet Halley?*

(b) *What is the closest and farthest distance from the Sun to Comet Halley over the course of the comet's orbit?*

(c) *Comet Halley is typically only visible for several months out of the 76 years of its orbit. Use Kepler's laws to explain why.*

(d) *When should we expect to see Comet Halley again?*

(a) The semi-major axis is the same as the average distance of the comet from the Sun, and is the A in $P^2 = A^3$. Knowing that $P = 76$ years, we can figure out A:

$$A \;=\; P^{2/3} \;=\; 76^{2/3}\,\mathrm{AU} \;=\; 18\,AU$$

(b) For this, you need the eccentricity of Halley's orbit, which is $e = 0.967$. (This was sent out to the class mailing list.) Using this, the closest approach is $A(1-e)$, or $(18\,\mathrm{AU})(1-0.967)$, which is $\boxed{0.59\,\mathrm{AU}}$. The farthest is $A(1+e)$, or $\boxed{35\,\mathrm{AU}}$.

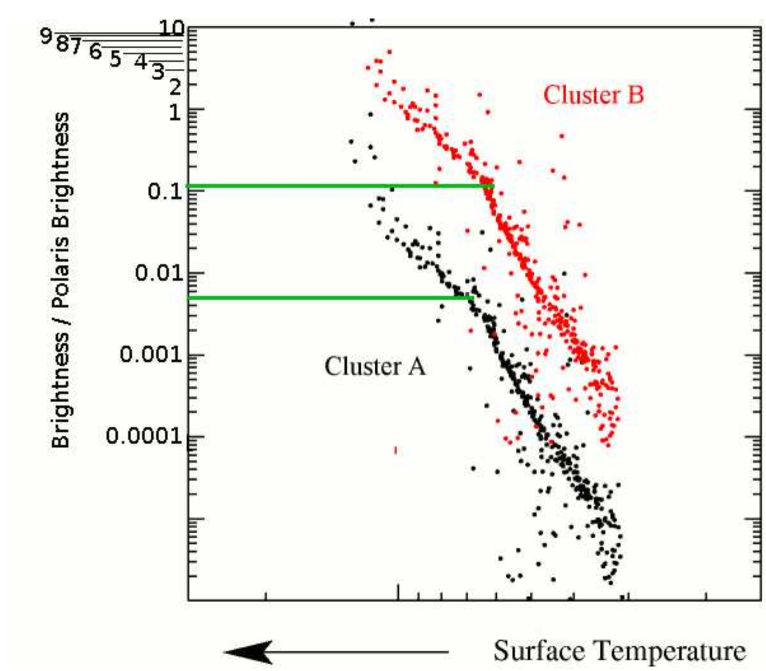
(c) Kepler's third law tells us that an object orbiting the Sun moves *faster* when it is closer to the Sun. 18 AU is a good long way out, at which distance it is difficult to see a comet. We're really only going to see the comet well when it is closer to 1 AU away from the Sun, where the Earth is. However, this is when the comet is moving the fastest, so it gets through this closer part of its orbit in much less time than it gets through the more distant part of its orbit. Thus, the comet is only close for a small fraction of the time it spends going around its entire elliptical orbit.

(d) 1986+76=2062. I don't know if I'll still be alive, but many of you will be.

5. *Consider the following H-R diagram of two star clusters:*

Each cluster is small enough that all of the stars in the cluster can be assumed to be at the same distance. Notice the log scaling of the "Brightness" axis. The tics from 1 to 10 are repeated in the other ranges; for example, the next tic above 0.1 represents 0.2, the tic above that 0.3, etc., even though they are not evenly spaced.

If Cluster A is at a distance of 1 kpc, how far away (in pc) is Cluster B? Be sure to explain your reasoning!



The two clusters are *only* showing the upper-left to lower-right strip that is characteristic of a main sequence; there seem to be few or no giants in this cluster. (We will later learn that this is a characteristic of a young cluster.) Since we know that the main sequence on the H-R diagram (when plotted

Luminosity vs. Temperature) shows up at specific luminosities for specific colors, we can figure out how far away a cluster is by looking at what brightness we get at a given temperature, figuring out the luminosity at that temperature, and using $B = L/(4\pi\, d^2)$. For a given star, this is fraught with peril, because it's difficult to know the luminosity of one star. For a cluster, though, we've got lots of stars and can identify the general trend of where the stars fall.

For these two clusters, we can pull out any point we want along the cluster to compare brightness. The "kink" is an easily identifiable feature that we identify here. Note that it's important to realize that we are comparing the brightnesses of *individual stars*, not of the entire cluster. Although these two clusters are (too) similar, real clusters will have different numbers of stars, affecting the overall brightness of the clusetr. However, this method still works. Also, the cluster doesn't have a single temperature; different stars in the cluster have different temperatures, as can be readily seen from the plot.

Stars at the kink in Cluster A look to be 0.005 times as bright Vega, and stars at the kink in Cluster B look to be 0.1 times as bright as Vega. Already, we know that Cluster B has to be closer, since stars are brighter than corresponding stars of the same time in cluster A. To actually compare, solve:

$$B \;=\; \frac{L}{4\pi\, d^2}$$

for luminosity:

$$L \;=\; B\, 4\pi\, d^2$$

Knowing that a star at the kink in each cluster has the same luminosity, we can set $L_A \;=\; L_B$, or:

$$B_A\, 4\pi\, {d_A}^2 \;=\; B_B\, r\pi\, {d_B}^2$$

where B_A and B_B are the brightnesses of a star at the kink in each cluster. Solving this for d_B, we get:

$$d_B \;=\; d_A \sqrt{\frac{B_A}{B_B}}$$

$$d_B \;=\; (1\,\mathrm{kpc})\sqrt{\frac{0.005}{0.1}}$$

This gives us a result of $\boxed{d_B \;=\; 220\,\mathrm{pc}}$.