Astronomy 102, Fall 2004

Homework Set 6 Solutions

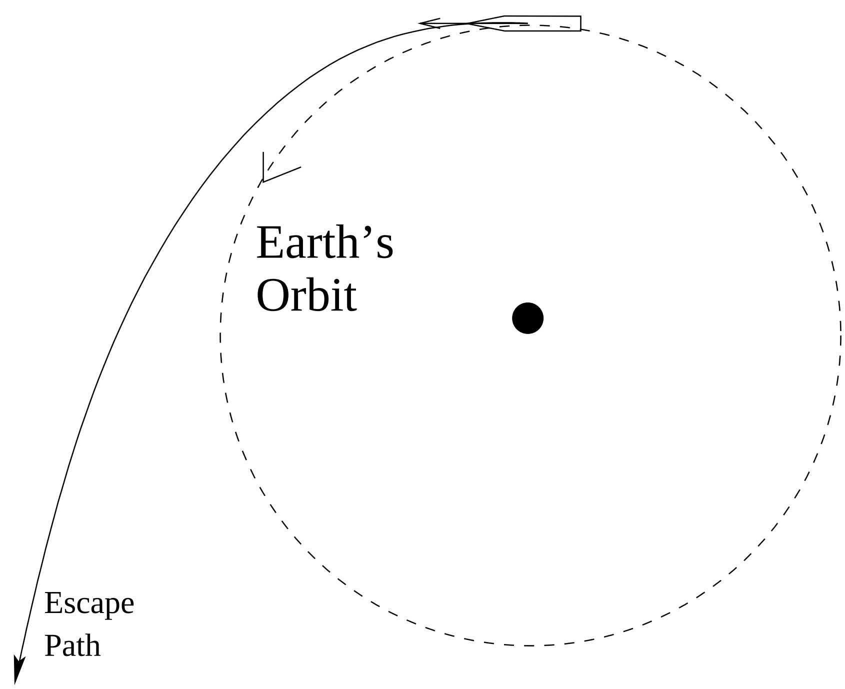
1. *You want to escape from the Solar System! Waaaah! You strap yourself into your rocket and blast off from the vicinity of the Earth. You've already managed to escape from the Earth (but are still travelling along in the direction of Earth's orbit), and now only need to break free of the gravitational grip of the Sun. You can fire a single, short burst from your rocket to greatly increase, decrease, or change the direction of your velocity.*

In which direction do you want to point your rocket (blasting the fuel out of the back) so as to escape the Solar System while using the minimum possible amount of fuel? Do you want to point it towards the Sun, away from the Sun, in the same direction as Earth's motion about it's orbit, in the direction opposite Earth's motion, or in some direction between one of these four "cardinal" directions? Note that escaping from the Sun means effectively putting yourself into an orbit that will take you a very, very long way from the Sun. Think about what we've talked about in terms of orbits and the conflict of gravity and motion, and justify the direction in which you want to point your rocket.

What matters is your total *speed*. Consider gravity vs. motion: you need more motion if you want to win out over gravity.

Alternatively, consider the elliptical orbits of Kepler's law (the "equal areas" law in particular). At 1 AU from the Sun, the greater your speed along your orbit, the greater the eccentricity of your elliptical orbit, and therefore the farther away the far side of the orbit. What you want is an elliptical orbit where the far side is infinitely far away. . . or an eccentricity of 1.

Either consideration indicates that what really matters is your net *speed*. Therefore, the most efficient thing is to take advantage of the speed you already have, and accelerate in the same direction as you're already moving. That way, you don't waste any of your fuel turning, but rather use it all to increase your speed. Your path will *still* be curved by the gravity of the Sun, but if you have enough speed, the gravity won't be enough to hold you in.



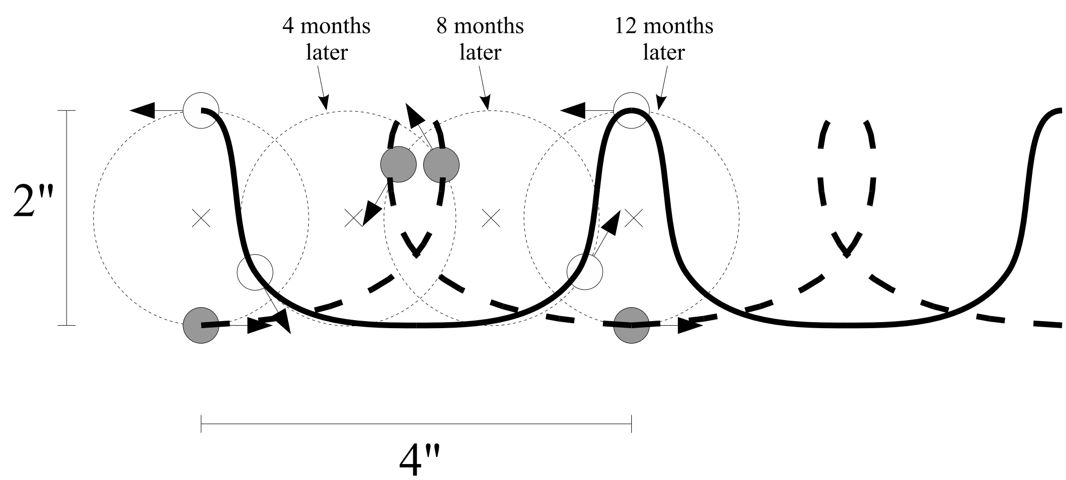
2. *Is it easier to find planets around stars whose orbits are "face-on" (i.e. you are looking "down" on the plane of the orbit) or "edge-on" (i.e. you are looking at "the side" of the plane of the orbit)? Why?*

Edge-on. Most extrasolar planets were discovered using the Doppler effect to observe the very small reflex motion of the star as the planet orbits the star. As the planet tugs first one way then the other, the star wobbles back and forth. If the system is not face-on, sometimes that wobble will be coming towards you, sometimes it will be going away from you, which you can observe as a period redshift and blueshift. The closer the system is to edge-on, the greater the fraction of the wobble there will be along the line of sight, and thus the easier it will be to see this periodic redshift and blueshift.

3. *You discover a binary star system that has a high proper motion. One of the stars is much brighter than the other one2014 the difference is enough that in the telescope you're using, you can only see the brighter star. You're observing the orbits of the binary star system "face-on" (all of the motion of the orbit is in the plane of the sky). The period of the binary star orbit is about a year. The stars are separated by about 2", and the system has a proper motion of about 4"/year.*

If you obseve the binary star system for several years, make a sketch of the path across the sky that you would observe the visible star to follow (using a solid line) and that you would observe the fainter star to follow (using a dashed line).

Your exact answer will depend on what you assume about the masses of the two stars. To keep the separation of the stars constant, assume that they're in circular orbits. Let's also simplify it by assuming that the masses of the stars are the same. (The brighter one may be a red giant, and the dimmer one a white dwarf, both of the same mass.) Then, the paths you get (over two years, which the OED tells us is included in the definition of "several") could look something like the following:



4. *In class, we discussed the rotation curve of the Milky Way (and other spiral galaxies) as evidence for dark matter. Similarly, many (though perhaps not all!) elliptical galaxies show evidence for dark matter, and we can find evidence for dark matter in clusters of galaxies. Since these systems consist of objects (stars or galaxies) randomly moving about in all directions, and don't have an ordered rotation, the same "rotation curve" arguments we used for our Galaxy won't apply. Instead, we talk about velocity dispersion: the range of velocities (measured from the Doppler shift) that galaxies in a cluster show. Consider two hypothetical clusters of galaxies, each 10kpc in diameter, and each with an identical number of galaxies of identical brightnesses. One of these hypothetical clusters (from our Universe) has dark matter, but the other has absolutely none. Which cluster will show a greater range of velocities? (Hint: remember that these clusters are held together by gravity, and that their size is dominated by how far galaxies can get before they are pulled back to the cluster. Also think about what would happen if you dropped a ball on the Earth compared to dropping one on the Moon (where gravity is lower), or what would be different if you wanted to throw a ball to a height of 10m on Earth as compared to throwing a ball to a height of 10m on the Moon.)*

The cluster with dark matter will show a greater range of velocities (a greater velocity dispersion). Simply in terms of gravity vs. motion, where there's more gravity, you can have more motion without flying apart. Or, in order for the galaxies to be able to move far enough apart from each other to make the cluster 10kpc in diameter, you will *need* more motion on the average to let them "fluff up" that much against the gravity holding them together.

5. Chapter 16, Question 10: *How does the shape of the Milky Way Galaxy's rotation curve imply the presence of dark matter, given the distribution of visible light in the galaxy?*

The visible light in the galaxy is spread out, but is more concentrated in the center than it is farther out. Because it's spread out, you don't expect the rotation curve to drop off as fast as it does in the solar system. Farther out, there is more mass pulling the stars toward the center of the galaxy than there is closer in. In contrast, in the solar system, all the planets have the same amount of mass (the Sun) pulling them towards the center. Therefore, you'd expect the stars in the galaxy to orbit faster than they would if all the mass were right at the center. However, given how the Galaxy's visible mass drops off with distance, you do expect the rotation curve also to drop off (lower velocities at greater distances).

In contrast, the rotation curve is observed to be approximately flat, even out to very large distances. Stars and gas are orbiting the Galaxy too fast for the amount of mass that we can see— they ought to fly away from the Galaxy. Since they don't, this tells us that there must be more mass holding them in than we can see. This mass we can't see is what we call dark matter.