Astronomy 102, Fall 2004

Homework Set 7 Solutions

1. *How many kg of mass is the Sun losing each second by converting mass into energy?*

The Sun's luminosity is 3.85×10^{26} W, which means it puts out 3.85×10^{26} J of energy each second. All of this comes from mass which is converted to energy, so we can figure out the amount of mass that must be converted using $E = mc^2$:

$$3.85 \times 10^{26}\,\mathrm{J} = m\,(3 \times 10^{8}\,\mathrm{m\,s^{-1}})^2)$$
$$m = \frac{3.85 \times 10^{26}\,\mathrm{kg\,m^2\,s^{-2}}}{(3 \times 10^{8}\,\mathrm{m\,s^{-1}})^2}$$

This tells us that $\boxed{4.28 \times 10^{9}\,\mathrm{kg}}$ (4.28 billion kg) of mass gets converted to energy by the Sun each second. Yikers!

2. (a) *Hydrogen fusion has an efficiency of about 0.7% for converting mass into energy. Assume that the Sun will use 10% of its Hydrogen for fusion (and that it is mostly Hydrogen). Given the Sun's luminosity, how long will it shine? **

(b) *We know that the Sun is not "on fire" because chemical reactions are not nearly efficient enough to keep the Sun shining at its current luminosity for anything like the amount of time we know it's been around. The efficiency of chemical burning of Hydrogen in the reaction $2H_2 + O_2 \longrightarrow 2H_2O$ is about 2×10^{-10}: that is the fraction of its mass that gets converted into energy. Assuming that the Sun were made up of Oxygen and Hydrogen in just the right proportions, and that it was able to burn all of its mass, how long (for how many years) would it be able to shine at its current luminosity using chemical reactions?*

(c) *Another possible energy source is the energy released from gravitational contraction. When you drop something from a height, energy is released; you may use that energy to make a sound, break something, etc. Suppose that you consider all the mass of the Sun to have been "dropped" from a great distance on to the Sun. The total energy released is approximately $G\,M^2/R$, where M is the mass of the Sun and R is the radius of the Sun. If this were where the Sun got its energy, for how long would it be able to shine at its current luminosity?*

(a) The mass of the Sun is $2.0 \times 10^{30}\,kg$; 10% of this is $2.0 \times 10^29\,\mathrm{kg}$. If 0.7% of this gets converted into energy, then the total amount of mass converted to energy is:

$$(0.007)\,(2.0 \times 10^{29}\,\mathrm{kg}) = 1.60 \times 10^{27}\,\mathrm{kg}$$

The total amount of Energy that the Sun will make over it's lifetime can be figured out from $E = m\,c^2$:

$$E = (1.60 \times 10^{27}\,\mathrm{kg})\,(3.0 \times 10^{8}\,\mathrm{m\,s^{-1}})^2$$
$$E = 1.44 \times 10^{44}\,\mathrm{J}$$

The rate that the Sun is using this up is $L_\odot = 3.85 \times 10^{26}\,\mathrm{J\,s^{-1}}$. The time it will live is the total energy being used up divided by the rate it is using up energy. (If you have 10 J to burn, and are burning 2 J each second, you will burn for 5 seconds; 10/2=5.)

$$t_\odot = \frac{1.44 \times 10^{44}\,\mathrm{J}}{3.85 \times 10^{26}\,\mathrm{J\,s^{-1}}}$$

$$\boxed{t_\odot = 3.7 \times 10^{17}\,\mathrm{s} = 10 \times 10^{9}\,\mathrm{years}}$$

Unsurprisingly, this is the same as the lifetime of the Sun. . . It will die when it's used up the fuel it's got. **Note** that this answer should only have one significant figure! The 0.7% figure is good to only one significant figure, as is the 10% estimate.

(b) This is exactly the same calculation as in part (a), only the 0.007 efficiency has been replaced with 2×10^{-10}, and the 10% mass has been replaced by 100% of the mass:

$$t_\odot = \frac{(2 \times 10^{-10})\,(2.00 \times 10^{30}\,\mathrm{kg})\,(3.0 \times 10^{8}\,\mathrm{m\,s^{-1}})^2}{3.85 \times 10^{26}\,\mathrm{J\,s^{-1}}}$$

$$\boxed{t_\odot = 9.4 \times 10^{10}\,\mathrm{s} = 3,000\,\mathrm{years}}$$

The historical record makes it clear that the Sun has been shining many times longer than this! This is why we know that chemical reactions are insufficient to power the Sun. Again, there's only one significant figure here.

(c) A similar calculation, except that the total energy available is:

$$E = \frac{G\,M^2}{R} = \frac{(6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}})\,(2.0 \times 10^{30}\,\mathrm{kg})^2}{6.96 \times 10^{8}\,\mathrm{m}} = 3.83 \times 10^{41}\,\mathrm{J}$$

The total time to shine is then:

$$t_\odot = \frac{3.83 \times 10^{41}\,\mathrm{J}}{3.85 \times 10^{26}\,\mathrm{J\,s^{-1}}}$$

$$\boxed{t_\odot = 1.0 \times 10^{15}\,\mathrm{s} = 32\,\mathrm{million\ years}}$$

The geologic record makes it clear that the Solar system has been around for billions of years, so gravitational potential energy from the contraction of the Sun to its current size is insufficient to power the Sun.

3. *Right now the Sun is a main-sequence star. Later in its life, it will become a red giant: it's luminosity will go up hundreds or thousands of times, it will be many (between ten and a hundred) times larger, and it will be a bit cooler.*

(a) Will the rate at which the Sun converts some of its mass to energy be higher or lower when it is a red giant as compared to now? Why?

(b) Do you expect that the pressure near the surface, but inside the Sun, will be higher when the Sun is a red giant than it is now? Why?

(c) The "solar wind" is a stream of particles coming off of the surface of the Sun. Do you expect the solar wind to be more intense (i.e. more particles coming off of the Sun per second) when it is a red giant or now? Why?

(a) Higher. The luminosity will be higher; all that energy comes from conversion of mass to energy, so therefore the mass conversion rate may be higher.

(If by "why" you read "what physically makes the rate higher" rather than "how do you know", the answer is that the degenerate helium core's gravity pulls a much shell of Hydrogen around the core that is much denser than the present core of the Sun. The higher density leads to a higher rate of fusion.)

(b) Gotta be a lot lower. Two ways to think about this. First, the temperature is lower (red rather than yellow), which means the particles are each moving slower. Second, the average density of the star is much lower, since the same mass of particles is spread over a much larger volume. (The core is actually denser, but the envelope is less dense.) A lower density of particles moving slower will mean a lower pressure. (Or think $P = (N/V)\,RT$; T is temperature, N/V is particle density.)

The second way to think about this is to realize that it is the balance of gravity and pressure that holds the star up (hydrostatic equilibrium). The mass of the star is the same, but the outer parts of a red giant are much farther from the core than is the case with a main sequence star. As the force of gravity gets weaker with distance, gravity on the outer parts of the star will be weaker in the red giant, so the pressure doesn't need to be as strong in order to hold the star up.

(c) Much higher. First, there's a greater surface area from which particles can come off. Most importantly, however, is the fact that the gravity at the surface of the star is much less (see (b) above). This means that the particles aren't bound as tightly. They are also moving slower (because the temperature is lower), but the temperature difference is only a factor of about 2, whereas the radius is maybe 100 times larger, so gravity is 10,000 times weaker.

4. *Consider a two large clouds of gas with solar abundances (i.e. mostly Hydrogen and Helium, with just a little bit of other things mixed in). Take one cloud of this gas, and make sun-like stars out of it. Let the sun-like stars live their main sequence lifetimes, and then (somehow, magically) disperse those sun like stars back into a cloud of gas. Qualitatively, how will the relative elemental abundances in the cloud which was processed through sun-like stars compare to the cloud which just remained a large, quiet cloud of gas?*

The cloud which was processed through stars will have a greater fraction of Helium than Hydrogen, but will still mostly be Hydrogen. The Sun-like stars will fuse something like 10% of their mass of Hydrogen into Helium.