Astronomy 102, Fall 2004

Homework Set 8 Solutions

1. *(a) A core-collapse supernova happens in our Galaxy about once every 100 years. Approximately how many neutron stars should there be in our galaxy?*

(b) Should the number of white dwarves in our galaxy be much greater, greater, similar, or smaller than the number of neutron stars? Explain.

(a) The galaxy has been around for about 10^{10} years. If a neutron star is produced every 10^2 years, then there should be $\boxed{10^{10}/10^2 = 10^8}$ neutron stars (that's 100 million neutron stars!) in our galaxy. Note that some supernovae produce black holes rather than neutron stars, but they are only the most massive stars, which are the rarest sorts. The number of black holes should be quite a bit less than the number of neutron stars, and will not affect this order-of-magnitude estimate.

(b) Much larger. The highest mass stars that go supernova are the rarest stars. There's been plenty of time for many lower mass stars to live through their lives and become white dwarves; because those lower mass stars are more common, there will be many more white dwarves left behind.

2. *(a) In a core-collapse (type II) supernova, where does all of this energy come from?*

(b) In a thermonuclear (type Ia) supernova, where does all of this energy come from?

(Optional, 1-point extra credit) Do the calculations (based on numbers and equations you know from lecture and other material in this course) to show that roughly the right amount of energy is released for each of the mechanisms you identified in (a) and (b).

(a) See the homework from last week, in particular problem 2c. There is energy released when you bring things closer together. In a core collapse supernova, the core collapses— it is about the mass of the Sun, and collapses from being about the radius of the Earth to a mere 10km or so in radius. A huge quantity of gravitational energy is released in that collapse, and that's what powers the supernova. So, this type of supernova is well-named; the very name gives you a hint as to where all the energy is coming form.

(b) In a thermonuclear supernova, all the energy comes from the fusion of Carbon and Oxygen into heavier elements (all the way up to elements like Iron). Once again, the supernova is well-named.

(c) In a core-collapse supernova, the difference in gravitational energy before and after is:

$$\Delta E = \frac{G\,M^2}{R_{\mathrm{after}}} - \frac{G\,M^2}{R_{\mathrm{before}}}$$

M is going to be 1.4 times the mass of the Sun, or 2.8×10^{30} kg. R_{after} is about 10 km, and R_{before} is about 6×10^6 m, the radius of the Earth. (Actually, less than that, but we're doing order of magnitude here, so this estimate is OK.)

$$\Delta E = (6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}})\,(2.8 \times 10^{30}\,\mathrm{kg})^2\left(\frac{1}{10^4\,\mathrm{m}} - \frac{1}{6 \times 10^6\,\mathrm{m}}\right)$$

$$\Delta E = 6 \times 10^{46}\,\mathrm{J}$$

The energy released by a supernova is of order 10^{46} J, so the calculation shows that indeed the gravitational potential energy that you get out of the collapse is roughly the right amount!

For the thermonuclear supernova, you have to know the efficiency of fusing Carbon all the way up to Iron. This is a little trickier based on everything we've done in this class. Let's try handwaving it. We know it's going to be less efficient than Hydrogen fusion, but probably not a whole heck of a lot less. We know Hydrogen fusion is 0.7% efficient, so let's *guess* 0.1% efficiency for fusing

Carbon all the way up to iron. We probably won't be wrong by more than a factor of 10, so we're probably going to be OK for an order-of-magnitude estimate!

Given that, the mass converted to energy is going to be about 0.001 times the mass of the white dwarf that blows up, or $1.4\,M_\odot$. Use $E = m\,c^2$ to figure out how much energy comes out.

$$E = (0.001)\,(2.8 \times 10^{30}\,\mathrm{kg})\,(3 \times 10^{8}\,\mathrm{m\,s^{-1}})^2$$

$$E = 3 \times 10^{44}\,\mathrm{J}$$

Sounds OK to order of magnitude. (Yes, a core-collapse supernova does release more energy, but a thermonuclear supernova is actually brighter. How to resolve this apparent contradiction? The answer is that a thermonuclear supernova puts out more *light*. Most of the energy of a Type II supernova actually comes out in the neutrinos produced.)

3. *A cluster of stars formed sometime in the past somewhere in our Galaxy. Typically, when clusters form, the number of stars that form with each mass (i.e. how many 20Msun stars, how many 1Msun stars, how many 0.5Msun stars, etc.) is the same as what is found in young stellar populations anywhere in our galaxy.*

Stars spend about 10% of their life as giants.

If you find and look at all the stars in this cluster now, what fraction of the stars do you think will be red giants? (I.e. much more than 10%, more than 10%, about 10%, less than 10%, much less than 10%.) Explain.

Much less than 10%. Most of the stars made when stars form are low mass stars, less than the mass of the Sun. Stars less than about $0.8\,M_\odot$ have a main sequence lifetime which is longer than the age of the Universe, so none of them have reached the red giant stage yet.

4. *When you look at a globular cluster through a small telescope with your eye, the individual stars you can see are largely red giants.*

(a) Why is this so?

(b) Why aren't you seeing any high-luminosity massive main sequence stars?

(c) Is what you are seeing representative of the population of stars in the globular cluster? Why or why not?

(a) Because the red giants are the most luminous stars in the cluster. Look at an H-R diagram for a globular cluster; the stars with the highest luminosity are all over on the giant branch. The stars in the cluster are all very close to the same distance from us, so the brightest ones will be the most luminous ones.

(b) Because globular clusters are all at least 10 billion years old, which is much longer than the lifetime of a high-mass star.

(c) No. Most of the stars in the globular cluster will be much dimmer low-mass main sequence stars, which we don't see in a small telescope simply because they're too dim.