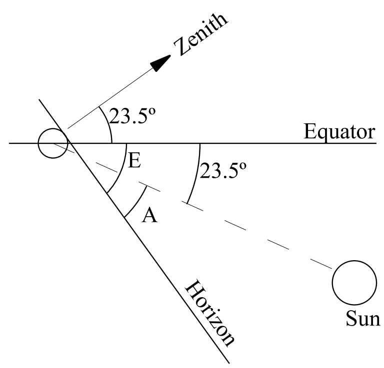
Astronomy 102, Fall 2004

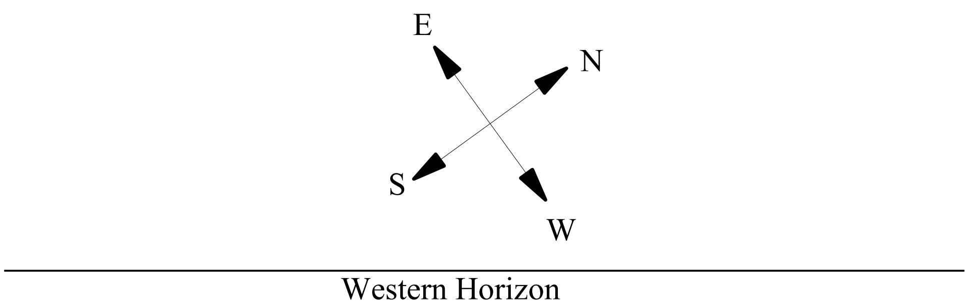
Review Exam 1 Solutions

**1.** On December 22, the Sun is at a declination of -23.5°, or 23.5° below the horizon. If you are at a latitude of 23.5° north, then your zenith is at declination +23.5°, or 23.5° ablove the equator. Both of these angles are shown in the diagram below:




The angle E is the altitude of the equator off of your horizon: E+23.5°=90°, so E=66.5°. The Sun is 23.5° lower than the equator, so the Sun's altitude is $66.5° - 23.5°$, or $\boxed{43°}$.

**2.**




**3.** If the moon is 45° up (that's halfway up) in the west, and is at first quarter phase (meaning a half moon after new moon), the Sun should be 90° further west than the moon. That means the Sun is roughly 45° below the western horizon. That's 1/8 of the way around the sky, so the Sun set 3 hours ago. That makes it probably something like 9PM.

**4.** It is *not* realistic. To have the crescent moon as drawn, you'd have to have the Sun a bit higher in the sky than where the moon is. However, this is a nighttime scene. To have a crescent moon over the horizon at night, the Sun must be just below that same horizon. Then, the lit side of the moon would be the "bottom" side, closer to the horizon, in contrast to what is drawn.

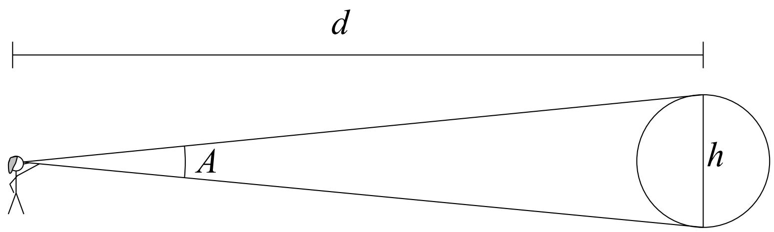
**5.**

**(a)** No. Alpha Centauri is so way the heck farther away than Pluto is that you're only about one ten-thousandth of the way there (if Pluto were even in the right direction, which it's not) once you reach Pluto. It would be akin to your first few steps on the way walking to Memphis.

**(b)** No. Alpha Centauri is about a parsec away; the other side of the galaxy is tens of thousands of parsecs away. Once again, you haven't gone an appreciable fraction of the way.

**(c)** In this case, you've probably completed about 1/25 of the trip (800,000 pc divided by 30,000 pc). This is not a large fraction, but it becomes an appreciable fraction.

**6.** In the diagram below, $h$ is the physical diameter of Betelgeuse, $d$ is the distance to Betelgeuse, and $A$ is the angular diameter of Betelgeuse:




The diameter of Betelgeuse is 500 times that of the sun:

$$h = (500) \times (2 \times 6.96 \times 10^5\,\text{km}) = 6.96 \times 10^8\,\text{km}$$

Thus, the angular size of Betelgeuse in radians is:

$$A = \left(\frac{6.96 \times 10^8\,\text{km}}{650\,\text{light} - \text{years}}\right)$$

We can't just divide the numbers on the right, though, because the units don't match. Looking on the front of the test, we have various conversions between units that we can stack in order to figure out the conversion of light-years to km:

$$A = \left(\frac{6.96 \times 10^8\,\text{km}}{650\,\text{light} - \text{years}}\right) \times \left(\frac{3.26\,\text{light} - \text{years}}{\text{pc}}\right) \times \left(\frac{1\,\text{pc}}{206,265\,\text{AU}}\right) \times \left(\frac{\text{AU}}{1.496 \times 10^8\,\text{km}}\right)$$

Now all the units cancel. Multiply the numbers together to get:

$$A = 1.13 \times 10^{-7}\ \text{radians}$$

We need to convert this to arcseconds:

$$A = (1.13 \times 10^{-7}\,\text{radians}) \times \left(\frac{206,265''}{\text{radian}}\right)$$

$$\boxed{A = 0.023''}$$

(Note that really we only know the answer to one significant figure, since there's only one significant figure in the "500 times the diameter of the Sun". I wouldn't take points off if you reported two, but if you reported more than three I would penalize you, becase we *don't* know the answer that well.)

**7.**

**(a)** The total mass is $M = 30 \times 10^{12}\,\text{M}_\odot$. The volume is:

$$V = \frac{4}{3}\pi(1.5 \times 10^6, \text{pc})^3 = 1.4 \times 10^{19}\,\text{pc}^3$$

The density is the mass divided by the volume:

$$D = \frac{M}{V} = \frac{30 \times 10^{12}\,\text{M}_\odot}{1.4 \times 10^{19}\,\text{pc}^3}$$

$$\boxed{D = 2 \times 10^{-6}\ \frac{\text{M}_\odot}{\text{pc}^3}}$$

**(b)** The total mass is $M = 23 \times 0.6\,\mathrm{M_\odot}$, or $M = 13.8\,\mathrm{M_\odot}$. The volume is:

$$V = \frac{4}{3}\pi(3.5\,\mathrm{pc})^3 = 180\,\mathrm{pc}^3$$

The density is the mass divided by the volume:

$$D = \frac{M}{V} = \frac{13.8\,\mathrm{M_\odot}}{180\,\mathrm{pc}^3}$$

$$\boxed{D = 0.08\,\frac{\mathrm{M_\odot}}{180\,\mathrm{pc}^3}}$$

**(c)** Here's what seems odd. In class, we did the "extragalactic scales" tutorial, in which we figured out that galaxies are much closer to each other compared to their sizes than stars are.

But if galaxies are much closer to each other compared to their sizes, why is the density in a cluster of galaxies so much lower than the density of stars inside a galaxy?? If stars are *so* spaced out, shouldn't the density be low?

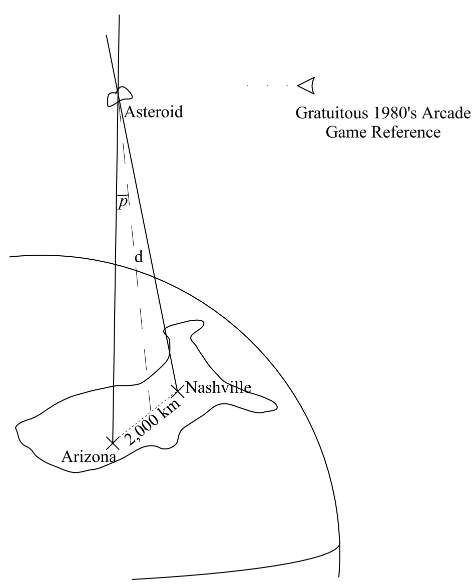
In fact, the size/spacing ratio doesn't tell you anything directly about the mass density of a collection of objects. Consider a box full of big fluffy cottonballs, with all the cottonballs packed close together. Even though the cottonballs are very close together compared to their size, the mass density is not very large.

On the other hand, consider filling the box *half* up with bullets, and then spreading those bullets out throughout the box. (Keep shaking the box to keep the bullets spread out, for example.) Even though the bullets are father apart compared to their size than the cotton balls were, there's more mass in the box now— and more mass in the box means greater mass density.

The point is that stars themselves are very dense concentrations of mass. Even though they are hugely spaced out within a galaxy, because each star is such a dense concentration of mass (on the scale of the Universe), by putting them together you get an appreciable mass density inside a galaxy. (That mass density is still *much* less than the density of air on Earth, however!) In between galaxies is almost *nothing*, however. So, even though galxies are spaced out by a distance only something like 10 times the size of a given galaxy, that's enough empty space to make the overall mass density inside a cluster of galaxies much lower than the density inside a galaxy.

**8.**

**(a)**



**(b)** The angle $p$ is small ($2''$ is a small angle), so we can use the formula:

$$p = \frac{h}{d}$$

In this case, we know $h$ (it's 1,000 km, or half of the baseline between Nashville and Arizona), and we want to find $d$, so we solve this equation:

$$d = \frac{h}{p}$$

To use this equation, though, we have to convert $p$ from arcseconds to radians:

$$p = (2'') \times \frac{1 \text{ radian}}{206,265''} = 9.7 \times 10^{-6} \text{ radians}$$

If we plug in this minimum angle we can measure, we'll get the maximum $d$ we can measure. (Any $d$ lower than that would give us a bigger angle, which would only be easier to measure with our precision.) This will give us $d$ in km (the same units as we have $h$ in).

$$d = \frac{1,000 \text{ km}}{9.7 \times 10^{-6}}$$

$$\boxed{d = 1 \times 10^{8} \text{ km}}$$

For purposes of comparison, convert this to AU:

$$d = 1.03 \times 10^{8} \text{ km} \times \left( \frac{1 \text{ AU}}{1.496 \times 10^{8} \text{ km}} \right) = \boxed{0.7 \text{ AU}}$$

**(c)** This is much larger than the distance from the Earth to the Moon, and is comparable to the distance from the Earth to the Sun.

**(d)** You will want it to be as high in the sky as it gets. If it's near rising or setting, the triangle will be "squished", giving you a smaller angle. Ideally, you want it to be just *past* its highest point in the sky as viewed from Nashville, and just *before* its highest point in the sky as viewed from Arizona, so that it's nicely positioned up between the two as drawn. Anywhere else, and you'll have to do some trig to take into account the "squished" triangle.