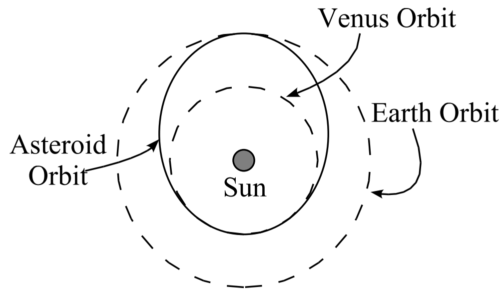
A102 Review Exam 2 Solutions

2004-October-8

1.

(a)



(b) The full major axis of the asteroid's orbit, from looking at the picture, is equal to the Earth-Sun distance plus the Venus-Sun distance, or 1.72 AU. The semi-major axis is half this, or $\boxed{0.76 \text{ AU}}$.

(c)

$$P^2 = A^3$$
$$P = A^{3/2} = (0.76)^{3/2} \text{ years}$$
$$\boxed{P = 0.66 \text{ years}}$$

2. From the parallaxes, we have distances:

$$d_A = \frac{1}{0.10}\,\text{pc} = 10\,\text{pc}$$
$$d_B = \frac{1}{0.025}\,\text{pc} = 40\,\text{pc}$$

We are also given a brightness ratio:

$$B_B = 2\,B_A$$

What we're looking for is luminosities. B is farther, but B is also brighter— so we'd better come out with B being more luminous! We expect a ratio $L_A/L_B < 1$.

We have distances, and we have brightnesses. We have distances and brightnesses, and know how to relate all that to luminosities:

$$B = \frac{L}{4\pi\,d^2}$$

Specifically, we have:

$$B_A = \frac{L_A}{4\pi\,{d_A}^2} \qquad B_B = \frac{L_B}{4\pi\,{d_B}^2}$$

What we want is a luminosity ratio, so solve each of these for luminosity:

$$L_A = 4\pi\,{d_A}^2\,B_A \qquad L_B = 4\pi\,{d_B}^2\,B_B$$

To get the ratio, divide those two puppies:

$$\frac{L_A}{L_B} = \frac{4\pi\,{d_A}^2\,B_A}{4\pi\,{d_B}^2\,B_B}$$

$$\frac{L_A}{L_B} = \left(\frac{d_A}{d_B}\right)^2 \left(\frac{B_A}{B_B}\right)$$

$$\frac{L_A}{L_B} = \left(\frac{10\,\mathrm{pc}}{40\,\mathrm{pc}}\right)^2 \left(\frac{1}{2}\right)$$

$$\boxed{\frac{L_A}{L_B} = \frac{1}{32}}$$

This does come out with B more luminous, as expected.

3. **(a)** The planets would go flying off away from the solar system into space, each continuing in the straight-line path in the direction that it was moving at the moment when the Sun vanished.

(b) They would continue in exactly the same orbits. The gravitational force depends on the mass of the two objects and the distance between them. The mass at the center of the Solar System is still the same, $1\,M_\odot$. The fact that it's a black hole rather than a G-type main sequence star doesn't change what the mass is, and thus doesn't change the gravitational effect on the planets.

4. **(a)**

$$\lambda f = c$$

$$f = \frac{c}{\lambda}$$

$$f = \frac{3.00 \times 10^8\,\mathrm{m\,s^{-1}}}{6563 \times 10^{-10}\,\mathrm{m}}$$

$$\boxed{f = 4.57 \times 10^{14}\,\mathrm{s^{-1}}}$$

(b)

$$E = h f$$

$$E = (6.626 \times 10^{-34}\,\mathrm{J\,s})(4.57 \times 10^{14}\,\mathrm{s^{-1}}$$

$$\boxed{E = 3.03 \times 10^{-19}\,\mathrm{J}}$$

(c)

$$(120\,\mathrm{J}) \left(\frac{1\,\mathrm{photon}}{3.03 \times 10^{-19}\,\mathrm{J}}\right)$$

$$\boxed{3.96 \times 10^{20}\,\mathrm{photons}}$$

(d)

$$\frac{\lambda_{\mathrm{obs}} - \lambda_{\mathrm{emit}}}{\lambda_{\mathrm{emit}}} = \frac{v}{c}$$

In this case, λ_{emit} is 6563Å. We need to find λ_{obs}:

$$\lambda_{\mathrm{obs}} - \lambda_{\mathrm{emit}} = (fracvc)\,(\lambda_{\mathrm{emit}})$$

$$\lambda_{\mathrm{obs}} = \lambda_{\mathrm{emit}} \left(1 + \frac{v}{c}\right)$$

We know c in m/s, not mph, so we have to convert:

$$c = 3.00 \times 10^8\,\frac{\mathrm{m}}{\mathrm{s}} \left(\frac{0.62\,\mathrm{mi}}{1,000\,\mathrm{m}}\right) \left(\frac{3,600\,\mathrm{s}}{1\,\mathrm{h}}\right)$$

$$c = 6.70 \times 10^8\,\mathrm{mph}$$

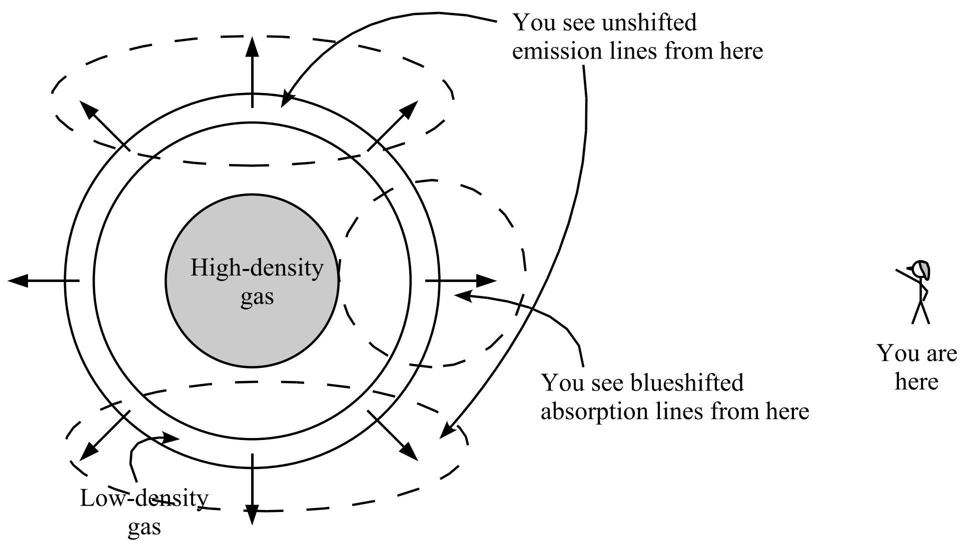
Now we can figure out the Doppler shifted wavelength. Note that the velocity we have is *negative* in this equation, because the ball is *approaching* us. We should get a blueshift, or a wavelength which is less than 6563Å.

$$\lambda_{\mathrm{obs}} = (6563\,\text{Å}) \left(1 + \frac{-90\,\mathrm{mph}}{6.70 \times 10^8\,\mathrm{mph}}\right)$$

$$\boxed{\lambda_{\mathrm{obs}} = 6563\text{Å}}$$

Is that a blueshift? Hard to say to this precision! 90mph is so bloody much less than the speed of light that it's very difficult to see the blueshfit at all.

5. **(a)**



(b) The line of sight direction from the hot continuum (blackbody) high-density source to you goes through the low density cloud. This part of the low density cloud will absorb some of the abundant photons from the high-density emitting source. Because the low-density cloud is moving *towards* you, you will get a blueshifted absorption line.

Meanwhile, the parts of the cloud off to the side also absorb some of the light from the continuum source in the middle. However, this is not along your line of sight, so you don't see that absorption line. The atoms *are* excited by the continuum source, and when they jump back down to the ground state they will emit photons in *all* directions. . . some of which will be towards you. Because this gas on the sides of the expanding cloud isn't moving towards or away from you, you will see this emitted light as an unshifted emission line.

Because the absorbed light on the side gets emitted in all directions, and only a little bit of that is towards you, you might expect the strength of the emission component of the P-Cygni profile to be weaker than that of the absorption component, as shown in the problem statement.

6. This one's a little bit hard. It's tempting to say "everyting else is the same, so the colors must be the same." While this is the right answer, it's an oversimplified explanation.

If the *angular* sizes are the same, then what you really know is that the size-to-distance ratio for the two stars is the same:

$$\frac{R_1}{d_1} = \frac{R_2}{d_2}$$

You also know the brightnesses are the same:

$$B_1 = B_2$$

So you know:

$$\frac{L_1}{4\pi\, {d_1}^2} = \frac{L_2}{4\pi\, {d_2}^2}$$

But you don't know how L_1 and L_2 compare, since you don't directly know how R_1 and R_2 compare.

What we're really after is temperatures, which relates to luminosity and size (two other things we've talked about) through:

$$L = 4\pi\, R^2\, \sigma\, T^4$$

Solve this for temperature, since it's what we care about:

$$T^4 = \frac{L}{4\pi\, \sigma\, R^2}$$

To see how the temperatures of the two stars compare, divide their temperatures; if $T_1 > T_2$, we'll know that Star 1 is bluer. If you can't figure out how the temperatures compare, then you can't say anything about colors!

$$\left(\frac{T_1}{T_2}\right)^4 = \frac{\frac{L_1}{4\pi\,\sigma\,{R_1}^2}}{\frac{L_2}{4\pi\,\sigma\,{R_2}^2}}$$

$$\left(\frac{T_1}{T_2}\right)^4 = \left(\frac{L_1}{4\pi\,\sigma\,{R_1}^2}\right)\left(\frac{4\pi\,\sigma\,{R_2}^2}{L_2}\right)$$

$$\left(\frac{T_1}{T_2}\right)^4 = \left(\frac{L_1}{L_2}\right)\left(\frac{R_2}{R_1}\right)^2$$

Hmm. We don't know the luminosity relationship, so it would seem we're stuck. Aha! We've got another relationship between luminosity and distance above from the brightnesses being equal; we can turn that into a ratio of luminosities with a little algebra:

$$\frac{L_1}{L_2} = \frac{4\pi\,{d_1}^2}{4\pi\,{d_2}^2}$$

$$\frac{L_1}{L_2} = \left(\frac{d_1}{d_2}\right)^2$$

OK. It's not clear that that helped, since we also don't know the distances... but we do know something about radii and distances together, from the angular sizes, so perhaps we've made progress. Let's try putting that in our temperature ratio equation:

$$\left(\frac{T_1}{T_2}\right)^4 = \left(\frac{d_1}{d_2}\right)^2\left(\frac{R_2}{R_1}\right)^2$$

Hey! Now we're making progress:

$$\left(\frac{T_1}{T_2}\right)^4 = \left(\frac{R_2/d_2}{R_1/d_1}\right)^2$$

But we already know that $R_2/d_2 = R_1/d_1$, so now we have that $T_1/T_2 = 1$, or $T_1 = T_2$. So we know the color of the stars is the same!

Be very careful though. We do *not* know that which star, if either, is more luminous, nor do we know which star is more distant! They could be at the same distance; or, one could be a yellow supergiant, whereas the other is a yellow main sequence star that's much closer (thereby balancing the angular size from being physically smaller and the brightness from being less luminous).

If you get a problem like this and don't know immediately how to do it, don't just freeze up and write down random equations. Explain what you know and what you need to figure out. If you're thinking along the right lines, you may get some partial credit. Also, by explaining that to yourself, you may actually be able to guide yourself through figuring out what it is that you need to do.

7. (a)

$$\lambda_{\text{max}} = \frac{2.9\times10^7\,\text{Å K}}{T} = \frac{2.9\times10^7\,\text{Å K}}{1500\,\text{K}}$$

So:

$$T = \frac{2.9\times10^7\,\text{Å K}}{\lambda_{\text{max}}} = \frac{2.9\times10^7\,\text{Å K}}{1500\,\text{Å}}$$

$$\boxed{T = 19,000\,\text{K}}$$

A star of a temperature like that is an O or a B star.

(a) Same calculation, on you get $\boxed{\text{T} = 3{,}700\,\text{K}}$. This is a K-type star.

(c) From the parallaxes, we know:

$$d_A = \frac{1}{0.0080}\,\text{pc} = 125\,\text{pc}$$

$$d_B = \frac{1}{0.080}\,\text{pc} = 12.5\,\text{pc}$$

We also have

$$B_A = B_B$$

So we know:

$$\frac{L_A}{4\pi\, {d_A}^2} = \frac{L_B}{4\pi\, {d_B}^2}$$

Solve this for the ratio of luminosities:

$$\frac{L_A}{L_B} = \frac{4\pi\, {d_A}^2}{4\pi\, {d_B}^2}$$

$$\frac{L_A}{L_B} = \left(\frac{d_A}{d_B}\right)^2 = \left(\frac{125\,\text{pc}}{12.5\,\text{pc}}\right)$$

$$\boxed{L_A/L_B = 100}$$

Sanity check: B is much closer, but has the same brightness as A, so B is dimmer. Good, that matches with what we got.

(d) If Star A is a main sequence B star, then just looking at where those stars fall on the H-R diagram, we can conclude that its luminosity is a bit less than 100 $L_\odot$. Therefore, the luminosity of Star B is a bit less than 1 $L_\odot$, from problem C. Star B is a K-type star; a K-type star that's of luminosity a bit less than 1 $L_\odot$ is much closer to the main sequence than it is to the giant branch, so it's probably a $\boxed{\text{main sequence star}}$.