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## Astronomy 102: Stars and Galaxies Review Exam 3

Instructions: Write your answers in the space provided; indicate clearly if you continue on the back of a page. No books, notes, or assistance from anyone is allowed. You are allowed to use, and will need, a calculator. The exam has six questions; four have equal weight, two (indicated) are valued as double problems.

This review test is too long, and you would probably be hard pressed to complete it in an hour. If you want to give it to yourself as a "simulated test," give yourself 75 minutes.

## $\underline{\text { Possibly Useful Constants and Formulae }}$

Earth-Sun Distance: 1.000 AU

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\begin{array}{cc}
1 \mathrm{pc}=206,265 \mathrm{AU}=3.086 \times 10^{16} \mathrm{~m} & \lambda_{\max }=\frac{2.9 \times 10^{7} \AA \mathrm{~K}}{T} \\
1 \mathrm{kpc}=10^{3} \mathrm{pc} \quad 1 \mathrm{Mpc}=10^{6} \mathrm{pc} & B=\frac{L}{4 \pi d^{2}} \\
1 \mathrm{~km}=1,000 \mathrm{~m}=0.62 \mathrm{miles} & F=\frac{G M_{1} M_{2}}{d^{2}} \\
1 \AA=10^{-10} \mathrm{~m} & V_{\text {sphere }}=\frac{4}{3} \pi \mathrm{r}^{3} \\
1 \text { year }=3.156 \times 10^{7} \mathrm{~s} & c=3.00 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1} \\
L_{\odot}=3.85 \times 10^{26} \mathrm{~W} & G=6.67 \times 10^{-11} \mathrm{~m}^{3} \mathrm{~kg}^{-1} \mathrm{~s}^{-2} \\
M_{\odot}=2.00 \times 10^{30} \mathrm{~kg} & 1 \mathrm{~W}=1 \mathrm{~J} \mathrm{~s}^{-1} \\
L_{\mathrm{Vega}}=55 L_{\odot} & P^{2}=A^{3} \\
d_{\text {Vega }}=7.8 \mathrm{pc} & P^{2}=\left(\frac{4 \pi^{2}}{G M}\right) A^{3} \\
\pi \text { radians }=180^{\circ} & z=\frac{\lambda_{\text {obs }}-\lambda_{\text {emit }}}{\lambda_{\text {emit }}}=\frac{\Delta \lambda}{\lambda} \\
206,265^{\prime \prime}=1 \mathrm{radian} \\
60^{\prime \prime}=1^{\prime} \quad 60^{\prime}=1^{\circ} & z=\frac{v}{c} \quad(\text { for } v \ll c) \\
A=\frac{h}{d} & (1+z)=\frac{\text { Size Now }}{\text { Size Then }}
\end{array}
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1. Discuss how the concept of gravity vs. motion applies in the following situations.
(a) The $P^{2}=A^{3}$ law in our solar system.
(b) The discovery of extra-solar planets. (Include an explanation of why considerations of gravity vs. motion made it easiest to discover large planets closest to the parent star, and how those systems need to be oriented.)
(c) Binary stars.
(d) The discovery of dark matter in galaxies.
(e) The accelerating expansion of the Universe.
2. Olber's Paradox is a famous "thought experiment" in cosmology. It states that if we live in an infinite, static Universe, every point on the sky - that is, every direction you can look - should look just like a point on the surface of the Sun. The reason is that in an infinite Universe, every line of sight will eventually intersect a star. For the Sun, lines of sight within about a $0.5^{\circ}$ angular diameter intersect a star. Most stars have a much smaller angular diameter (the farther the star, the smaller the angular diameter). If the Universe were infinite and static, however, eventually, however far away, every line of sight would intersect a star.
(a) In fact, this is not observed. You can look off to the side of a star and see blank, dark night sky. What does this tell us about our Universe?
(b) Give two ways in which the Big Bang theory helps explain why we don't see what Olber's Paradox suggests we should see.
3. The Cosmic Microwave Background (CMB) is observed today as a blackbody spectrum with a temperature of 2.728 K . The Universe today is 1,089 times larger than it was when the light of the CMB was emitted. Compared to the age of the Universe, the time it took for the CMB to be emitted was very short; it's fair to approximate it as having been emitted all at once. (Note: be sure to show your work in all parts below.)
(a) What is the peak wavelength of the CMB spectrum today?
(b) What is the redshift $z$ for the light in the CMB?
(c) What was the peak wavelength of the CMB when it was emitted?
(d) What was the temperature of the CMB when it was emitted?
(e) We describe the CMB as the "surface of last scattering". Why is it that we can describe the CMB - which was emitted everywhere in the Universe all at once as being on a "surface" a constant distance away from us?
(f) At the time of emission, what was the luminosity emerging from one square meter of the surface of last scattering? How does this compare to the luminosity of one square meter of the Sun? $\left(\mathrm{T}_{\odot} \simeq 5800 \mathrm{~K}\right)$
4.     - Double Problem Although figuring out the Sun's orbit about the center of the Galaxy is complicated because the matter in the Galaxy is distributed rather than all being concentrated at the center, it's a reasonable approximation to say that it's orbital speed is the same as it would be if all the matter closer to the center were concentrated right at the center:

(a) The Sun takes 250 million years to go around the center of the Galaxy. In our approximation, this is a Keplerian orbit. How much mass, including both normal and dark matter, (in both $M_{\odot}$ and kg ) is there closer to the center of the Galaxy?
(b) The Sun is 8 kpc from the center of the Galaxy. If you make the (not-very-good) approximation that that mass from (a) is distributed uniformly in a sphere, what is the mass density of the Galaxy (in $\mathrm{kg} / \mathrm{m}^{3}$ )? Compare this to the density of air (about $1 \mathrm{~kg} / \mathrm{m}^{3}$ ) or water $\left(1 \mathrm{~g} / \mathrm{cm}^{3}=1,000 \mathrm{~kg} / \mathrm{m}^{3}\right)$.
(c) Near the Sun, the densty of stars is 0.2 stars per $\mathrm{pc}^{3}$, and each star has an average mass of about $0.5 M_{\odot}$. What is the density of normal matter (most of which is in stars) near the Sun (in $M_{\odot} / \mathrm{pc}^{3}$ )?
(d) Convert your answer to (b) into $\mathrm{M}_{\odot} / \mathrm{pc}^{3}$. How does this compare to the density of normal matter near the Sun? What is the density of dark matter near the Sun?
(e) The Galaxy is $90 \%$ dark matter. Does this contradict your answer to (d)? If so, how do you resolve the contradiction?
5.     - Double Problem You find a cluster of galaxies. One of the galaxies in that cluster has the Hydrogen Alpha emission line (rest wavelength $6562.8 \AA$ ) observed at $6694.0 \AA$.
(a) If you were to ascribe the expansion of the Universe to galaxies flying away from us, what recessional velocity would you calculate for this galaxy?
(b) Use Hubble's Law to estimate the distance to the cluster of galaxies.
(c) A Cepheid Variable star in the galaxy is 1,200 times as luminous as the Sun. How bright would this star be observed to be in comparison to Vega? If a good telescope can detect stars observed to be $10^{11}$ times dimmer than Vega, would you be able to detect this star?
(d) By what percentage of its size did the Universe grow during the time it took the photons you observe to travel from the galaxy to you?
(e) If you estimate the mass of the cluster by summing up all the mass of the stars and gas that emit the light you should observe, and apply gravity vs. motion considerations, you would guess that galaxies in the cluster are moving about relative to each other at about $60 \mathrm{~km} / \mathrm{s}$. Knowing what you know about galaxy clusters, aboiut how fast are galaxies really seen to be moving relative to each other in a cluster? What is the reason for the discrepancy, if any?
(f) (You may find this question conceptually difficult....) Given your answer to (d), how precise is your answer to (b), assuming your measurement of the observed wavelength is perfect? That is, do you know the distance to within $0.1 \%, 1 \%$, $10 \%, 20 \%$, or within some other precision?
6. An astronomer observes a binary star system. He knows, somehow, that one star is more massive than the other. He also knows, somehow, that he's looking down on the plane of the orbit.
(a) The astronomer observes the stars orbit through one period. Each star moves at a constant speed (although the two don't necessarily move at the same speed as each other). On the diagram below draw the two paths through which each star moves, with an arrowhead to indicate dircton of motion. (The larger circle indicates the more massive star.)
(b) Now that the astronomer has observed the period, there is one more physical quantity about the binary stars that he would need to know in order to estimate the masses of the stars. What would that be? (Also indicate the units in which the astronomer might want to measure this physical quantity.)
(c) What two quantities can be measured by the astronomer on Earth in order to allow the astronomer to calculate the physical quantity in part (b)?
