A102 Review Exam 3 Solutions

2004-November-03

1. **(a)** As you get to larger and larger distances from the Sun (greater A), the gravitational force on a planet goes down. This means that the amount of motion needed to balance that gravity goes down, leading to lower planetary orbit speeds; between A and the speed, you can determine P. Notice that P goes up *faster* than proportional to A, whereas if the speed were constant (no conflict of gravity and motion) P would be proportional to A.

(b) We discovered most extrasolar planets by looking at the Doppler effect of the star being pulled around a very slight amount by its planet. More motion means more Doppler effect, which means it would be easier to find planets. You're going to get the most motion where the gravitational force of the planet is strongest, and that will happen for a massive planet closest to the star.

(c) Evidence for dark matter comes from rotation curves of galaxies. The stars and gas at large distances are moving too fast for the amount of gravity that should be there from the matter we can see. There must be additional (dark) matter to provide the additional gravity necessary to hold those stars and gas in to the galaxy.

(d) This is where the conflict of gravity and motion breaks down. Dark Energy has a *negative* gravitational effect, so rather than being in conflict with motion, gravity is making there be more and more motion all the time, leading to an accelerating expansion.

2. **(a)** One possibility is that the Universe is finite in extent. Eventually, you've seen far enough that you've seen past all of the stars. (This is true for our Galaxy, but turns out not to be true for the Universe.) The second possibility is that the Universe hasn't been around forever. Because light takes time to travel, it can only have gone so far in a Universe of a finiate age. This means that eventually you get to a distance great enough that light from stars at that distance won't have reached us yet.

(b) First is the finite age— the Big Bang gives us a beginning to the Universe. The second is the cosmological redshift. As you look farther back in time, the Universe has expanded more since the photons you see were emitted. This redshift moves them to longer wavelengths, and therefore lower energies. This reduces the energy you would detect from a very distant star or galaxy.

3. **(a)**

$$\begin{aligned}\lambda_{\mathrm{max}} &= \frac{2.9\times10^{7}\,\text{Å}\,\mathrm{K}}{T}\\ \lambda_{\mathrm{max}} &= \frac{2.9\times10^{7}\,\text{Å}\,\mathrm{K}}{2.728\,\mathrm{K}}\\ \boxed{\lambda_{\mathrm{max}} = 1.06\times10^{7}\,\text{Å}}\end{aligned}$$

(b)

$$\begin{aligned}1+z &= \frac{\text{Size Now}}{\text{Size Then}}\\ z &= \frac{\text{Size Now}}{\text{Size Then}} - 1\\ z &= 1089-1\\ \boxed{z = 1088}\end{aligned}$$

(c)

$$\begin{aligned}
\frac{\lambda_{\text{obs}} - \lambda_{\text{orig}}}{\lambda_{\text{orig}}} &= z \\
\frac{\lambda_{\text{obs}}}{\lambda_{\text{orig}}} - 1 &= z \\
\frac{\lambda_{\text{obs}}}{\lambda_{\text{orig}}} &= z + 1 \\
\lambda_{\text{orig}} &= \frac{\lambda_{\text{obs}}}{z+1} \\
\lambda_{\text{orig}} &= \frac{1.063 \times 10^{7}\,\text{Å}}{1089} \\
\boxed{\lambda_{\text{orig}} = 9,800\,\text{Å}}
\end{aligned}$$

(d)

$$\begin{aligned}
\lambda_{\text{max}} &= \frac{2.9 \times 10^{7}\,\text{Å}\,\text{K}}{T} \\
T &= \frac{2.9 \times 10^{7}\,\text{Å}\,\text{K}}{\lambda_{\text{max}}} \\
T &= \frac{2.9 \times 10^{7}\,\text{Å}\,\text{K}}{9762\,\text{Å}} \\
\boxed{T = 2970\,\text{K}}
\end{aligned}$$

Notice that this is less than the temperature of the Sun's surface!

(e) We're now detecting the light that's just reaching us from the emission of the CMB. Because light moves at a constant speed, that means that all of that light we're just now seeing must be coming from the same distance. (Anything emitted closer passed us by long ago, and anything emitted farther hasn't reached us yet.) All the points that are the same distance from us is a spherical shell around us— or a spherical "surface."

(f)

$$\begin{aligned}
L &= A\,\sigma\,T^{4} \\
L &= (1\,\text{m}^2)(5.67 \times 10^{-8}\,\frac{\text{W}}{\text{m}^2\text{K}^4})(2970)^4 \\
\boxed{L = 4.4 \times 10^{6}\,\text{W}}
\end{aligned}$$

$$\begin{aligned}
L_{\odot} &= (1\,\text{m}^2)(5.67 \times 10^{-8}\,\frac{\text{W}}{\text{m}^2\text{K}^4})(5800)^4 \\
L_{\odot} &= 6.4 \times 10^{7}\,\text{W}
\end{aligned}$$

A square on the surface of the Sun is 15 times more luminous than a square of the same size on the surface of last scattering was at time of emission. (Stars are hot!)

(4) (a) If the orbits are approximately Keplerian, we can use Kepler's laws. Specifically:

$$P^2 = \left(\frac{4\pi^2}{G\,M}\right) A^3$$

In this case, M is the mass at the center, and is what we want. Solve this equation for M:

$$M = \left(\frac{4\pi^2}{G}\right)\left(\frac{A^3}{P^2}\right)$$

Stick in the right numbers. The hardest part is making sure we got all the Units right.

$$M = \left(\frac{4\pi^2}{6.67\times10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}}\right)\left(\frac{(8\times10^3\,\mathrm{pc})^3}{(250\times10^6\,\mathrm{year})^2}\right)\left(\frac{3.086\times10^{16}\,\mathrm{m}}{1\,\mathrm{pc}}\right)^3\left(\frac{1\,\mathrm{year}}{3.156\times10^7\,\mathrm{s}}\right)^2$$

$$M = 1.4\times10^{41}\,\mathrm{kg}\left(\frac{1\,M_\odot}{2.00\times10^{30}\,\mathrm{kg}}\right)$$

$$\boxed{M = 1.4\times10^{41}\,\mathrm{kg} = 7.2\times10^{10}\,M_\odot}$$

(b) The volume is:

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\,(8\times10^3\,\mathrm{pc}^3)\left(\frac{3.086\times10^{16}\,\mathrm{m}}{1\,\mathrm{pc}}\right)^3$$

$$V = 6.3\times10^{61}\,\mathrm{m}^3$$

("It's a big galaxy, Mr. Scott." Identify the speaker for 10^{-7} points of extra credit.)
The density is then:

$$\frac{1.4\times10^{41}\,\mathrm{kg}}{6.3\times10^{61}\,\mathrm{m}^3} = \boxed{2.3\times10^{-21}\,kg/m^3}$$

(c) 0.2 stars per pc^3 with each star of mass half of a $M_\odot$ gives you a mass density in stars of $\boxed{0.1\,M_\odot/\mathrm{pc}^3}$

(d)

$$\left(2.27\times10^{-21}\,\frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(\frac{1\,M_\odot}{2.00\times10^{30}\,\mathrm{kg}}\right)\left(\frac{3.086\times10^{16}\,\mathrm{m}}{1\,\mathrm{pc}}\right)^3$$

$$\boxed{0.034\,M_\odot/\mathrm{pc}^3}$$

AARGH! I thought I'd fixed this before I posted the test. Guess not. This points out how bad the approximation is.... The real density is something between 0.1 and 0.5 $M_\odot/\mathrm{pc}^3$. You were *supposed* to get a bigger number (something like $0.14\,M_\odot/\mathrm{pc}^3$) in this part, and then comment that the extra was dark matter. This number wasn't going to be 10x bigger than the number from (c), however, and you were supposed to then comment that dark matter is more spread out in the Galaxy than stars, so here relatively close in there are more stars compared to dark matter than there are farther out.

But since the numbers didn't work out, you couldn't do any of that.

Oops.

Still, hopefully you can understand what we did with Kepler's third law above.

5. (a)

$$z = \frac{\lambda_{\rm obs} - \lambda_0}{\lambda_0} = \frac{v}{c}$$

$$v = c\left(\frac{\lambda_{\rm obs} - \lambda_0}{\lambda_0}\right)$$

$$v = \left(3.00 \times 10^5\,\frac{\rm km}{\rm s}\right)\left(\frac{6694.0\,\text{Å} - 6562.8\,\text{Å}}{6562.8\,\text{Å}}\right)$$

$$\boxed{v = 6.0 \times 10^3\,{\rm km/s}}$$

(b)

$$v = H_0\, d$$

$$d = \frac{v}{H_0} = \frac{5997\,{\rm km\,s^{-1}}}{71\,{\rm km\,s^{-1}\,Mpc^{-1}}}$$

$$\boxed{d = 84\,{\rm Mpc}}$$

(Yes, I will give you H_0 on the front of the test if you need it. Sorry about that for the review test.)

(c) We have $L_C = 1,200\,L_\odot$, and $L_V = 55\,L_\odot$.

$$B_C = \frac{L_C}{4\pi\,{d_C}^2}$$

$$B_V = \frac{L_V}{4\pi\,{d_V}^2}$$

Divide the two equations to get:

$$\frac{B_C}{B_V} = \frac{(L_C)\,(4\pi\,{d_V}^2)}{(L_V)\,(4\pi\,{d_C}^2)}$$

Cancel the 4π on the top and bottom, and plug in:

$$\frac{B_C}{B_V} = \frac{(1,200\,L_\odot)(7.8\,{\rm pc})^2}{(55\,L_\odot)(84.5 \times 10^6\,{\rm pc})^2}$$

$$\frac{B_C}{B_V} = 1.9 \times 10^{-13}$$

So, no, we couldn't detect this with our telescope.

(d) The time was travelling for a time $t = d/c$, where d is the distance and c is the speed of light. This time is:

$$t = \frac{d}{c} = \left(\frac{84.5 \times 10^6\,{\rm pc}}{1\ {\rm lyr\ yr^{-1}}}\right)\left(\frac{3.26\,{\rm lyr}}{1\,{\rm pc}}\right)$$

$$t = 275 \text{ million years}$$

During this time, the galaxy will have moved an additional distance away $\Delta d = vt$

$$\Delta d = \left(5997 \frac{\text{km}}{\text{s}}\right) (275 \times 10^6 \,\text{yr}) \left(\frac{3.156 \times 10^7 \,\text{s}}{1 \,\text{yr}}\right)$$

$$\Delta d = 5.20 \times 10^{19} \,\text{km} \left(\frac{10^{-6} \,\text{Mpc}}{3.086 \times 10^13 \,\text{km}}\right)$$

$$\Delta d = 1.69 \,\text{Mpc}$$

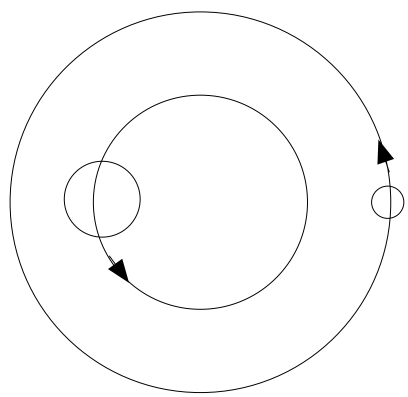
The percentange increase is then:

$$\frac{\Delta d}{d} = \frac{1.69 \,\text{Mpc}}{84.5 \,\text{Mpc}} = 0.020$$

The Universe's size increased by 2% in the last 275 million years. It doesn't seem to be expanding very fast after all!

Note that there was a much faster way to do this problem. Above, we had calculated a z; the answer wasn't written out, but that was the first thing we did to solve part (a). We would have gotten $z = 0.02$. Since we know $1 + z =$Size Now/Size Then, then we know that the Universe is 1.02 times the size it was when the light was emitted, for a 2% increase in size.

6. (a)



(b) Their physical separation, in units of AU or km or similar. Then he could use an equivalent of the $P^2 = (4\pi^2/GM)\, A^3$ law to figure out the masses. (We haven't actually gone through this calculation in class, so I won't make you do it.)

(c) The angular separation, and the distance to the binary star system. The latter could be measured with parallax, if the binary system is close enough.