Astronomy 102, Exam 1 Solutions 2006/09/15

1. (a) The two efficiencies are exactly the same, because the physical process involved is exactly the same.

A number of you said that because the blue star is producing more energy, it must be more efficient. This is wrong; it's just using the fuel faster. What's more, several of you said that the blue star must be 1,000 or 10,000 times more efficient. Given that the Sun's process has an efficiency of about 10^{-2} (from the front of the test, or 0.007 if you remembered the number from class), that would give the blue star an efficiency of $> 1\ldots$ which is, of course, impossible.

(b) The Sun will live longer. The blue star has 10 times as much fuel, but is using it 10,000 times faster (since it's generating energy at 10,000 times the rate with a process of the same effiency).

(c) Many of you made this problem harder than it needs to be. The blue star has 10 times as much fuel, but is using it 10,000 times faster, so it will live only 1/1,000 times as long as the Sun, or 10^7 years.

(d) They will have used the same fraction. Each star will have used 10%, or 0.1, of its mass as fuel, and about $10^{-2} = 0.01$ of that fuel's mass will have been converted to energy; thus, each star will have converted $(0.1)(0.01) = 0.001$ of its mass to energy.

A number of you said that each star would have converted 10% of its mass to energy. This isn't right; this is how much get used as fuel, but the energy generation process isn't perfectly efficient. A number of you also made the mistake of saying that $10^{-2} = 0.1$, which is wrong; $10^{-2} = 0.01$ (move the decimal place over twice).

2. Most of you got this right. As time goes by, more and more K-40 decays to Ar-40. Thus, a *lower* K-40/Ar-40 ratio (or, equivalently, a *higher* Ar-40/K-40 ratio) indicates that more K-40 has decayed, and thus more time has elapsed. The other system is older.

3. (a) Very likely. 10 half-lives will have elapsed, so the chance of the atom remaining *undecayed* is the same as the chance of flipping heads 10 times in a row — which works out to about one chance in a thousand, although I didn't expect you to calculate it.

(b) It's either there, or not there, and there's a 0.5 chance of each. You can't have part of a single isotope, so you can't have a "partially decayed" or "50% decayed" isotope.

(c) 0.5. The chance of an isotope decaying during a time period equal to its half life is 1/2, just as the chance of flipping heads each and every time you flip a coin is 1/2, regardless of what has gone before or what will come after.

4. (a) 1:1. One half-life has elapsed, so half of the K-40 has decayed to Ar-40, leaving equal amounts of both.

(b) 1:2. When the rock liquifies and re-solidifies, the Ar-40 will boil off, and won't recombine into rock, but the K-40 will. However, half of the K-40 that was in the rock to start with has already decayed, and the process of liquification and resolidification doesn't recreate new K-40.

(c) 1:1. One half-life has elapsed since the last time the rock liquified, effectively "resetting" the K-40/Ar-40 clock. Since there was *no* Ar-40 after the resolidification, the only Ar-40 that is there is what has been produced through K-40 decay.

(d) 1:4. Whether or not the K-40 decays isn't dependent on whether the rock is solid or liquid. Two half-lives have passed, so 3/4 of the K-40 has decayed.

NOTE: A number of you seemed to think that after the rock was liquified and re-solidified, the amount of K-40 was reset back to the beginning value. This is *not* the case. Refer also back to the homework problem about the supernova, where some of you seemed to think that the decay didn't start until it was in a solidified rock. Radioactive decay always happens, regardless of where the particle is.

NOTE 2: Some of you referred to the rock as "pure K-40". It isn't; it's mostly carbon and iron, probably, depending on where it comes from. No matter what, though, K-40 is a *trace* element in all of the Solar System rocks. However, we're never talking about absolute amounts, but *ratios* of K-40 to Ar-40, or K-40 at one time to Ar-40 at another time. We could talk about what fraction of the class is under age 19, and take ratios of under-19 now to under-19 at the begining of class. That fraction would decay with time, but even on the first day of class we weren't a class of pure "under 19".