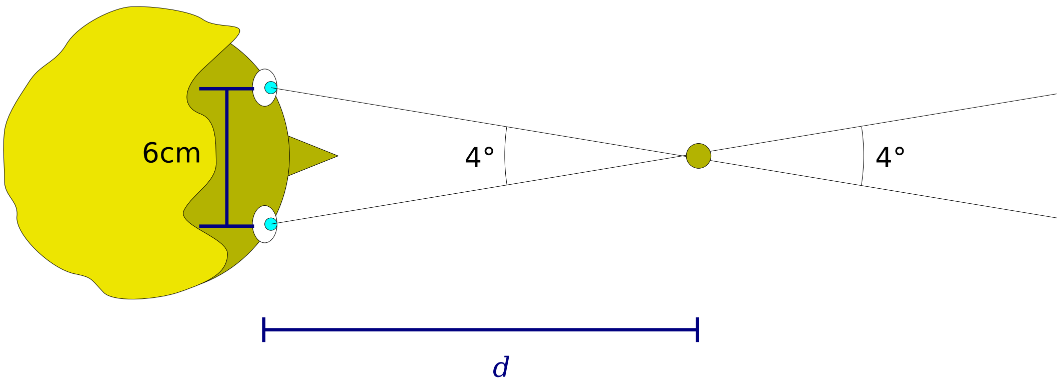
A102 Fall 2006 Exam 3 Solutions

1. The term "ladder" is appropriate for two reasons. First, as many of you noted, there are many different methods that tend to be useful at different distances. Thus, for greater distances, you have to use a method which is on a higher "rung." However, there is another crucial point, that is related to "climbing" the ladder : to use a method on a higher rung, you must calibrate it somehow from an already-calibrated method on a lower rung.

Parllax is the most direct distance measurement. It's just geometry. We have to know how big the AU is (the distance from the Earth to the Sun). Knowing that, we can measure the parallax angle (how much a star "wobbles" over the year) and directly calculate the distance. Unfortunately, this only works out to at most a kpc or so, well within our galaxy. However, if we can measure some Cepheid Variable stars with parallax (or another parallax-calibrated method such as main sequence fitting), with the distance and brightness of the Cepheid in hand we can calculate its luminosity. Knowing that, if we see another Cepheid, from its brightness we can calculate its distance.

Cepheid variables are good only out to relatively nearby galaxies. However, we have another standard candle: thermonuclear supernovae. If a few of those go off in galaxies that have Cepheid distances, we can figure out their luminosities, and use that knowledge to measure distances as far as we can see supernovae— which are very luminous, and thus visible very far.

2. **(a)**



(b) The easiest way to do this is to use the small angle formula. If you use the parallax formula directly, you'll get in trouble. . . because the baseline is not 1 AU! Many of you realized that the parallax angle here is 2 deg. Many of you who realized that also realized that if you use that, you have to use 3 cm as the baseline. Here, I'll just do the full triangle.

$$4\,\mathrm{deg}\left(\frac{\pi\,\mathrm{rad}}{180\,\mathrm{deg}}\right) = 0.0698\ \mathrm{rad}$$

Notice that I'm keeping extra digits beyond the number of significant figures for an intermediate calculation.

$$\frac{6\,\mathrm{cm}}{d} = 0.0698$$

$$d = \frac{6\,\mathrm{cm}}{0.0698} = \boxed{86\,\mathrm{cm}}$$

3. We have a Cepheid with luminosity $L_C = 30,000 L_\odot$. The dimmest one we can detect has brightness is $B_C = 10^{-11} B_V$. We also know the luminosity of Vega ($L_V = 130 L_\odot$) and the distance to Vega (7.76 pc), which we could use to calculate the brightness of Vega. . . although with just a wee bit of algebra, we can avoid ever calculating that number.

$$B_C = 10^{-11}\, B_V$$

$$\frac{L_C}{4\pi\, {d_C}^2} \;=\; 10^{-11} \frac{L_V}{4\pi\, {d_V}^2}$$

Solve this for what we care about, d_C:

$$d_C \;=\; d_V \sqrt{\frac{L_C}{10^{-11}\, L_V}}$$

Now we can put in numbers:

$$d_C \;=\; 7.76\,\mathrm{pc} \sqrt{\frac{30,000\, L_\odot}{(10^{-11})(130\, L_\odot)}}$$

$$d_C \;=\; 3.7 \times 10^7\ \mathrm{pc} \;=\; \boxed{37\,\mathrm{Mpc}}$$

That's quite a bit farther than M31. It's also farther than the Virgo cluster, although by less than a factor of 2. (Did you expect it to be between the two distances? Sometimes the Prof Psychology Theorem doesn't work. . . .)

These Cepheid distances won't get us out into the very distant Universe.

4. An 8 $M_\odot$ star only lives 40 million years. If it is already a giant, that means it's in about the last 10% of its life, and won't live more than 4 million additional years.

The distance to this galaxy is $37\,\mathrm{Mpc}\,(3.26\,\mathrm{lyr}/1\,\mathrm{pc}) \;=\; 120$ million light-years. Thus, the lookback time is 120 million years. That's longer than the whole lifetime of the 8 $M_\odot$ star, never mind the last 10% of it. That star is no longer a Cepheid variable; it's gone supernova by now, leaving behind a neutron star or a black hole.