A102 Fall 2006 Exam 4 Solutions

1. **(a)** A higher t_H means more time for the Universe to reach its current size, and thus a slower expansion rate. $\boxed{\text{Alan Sandage}}$ had the higher t_H.

(b) If a supernova is at a given redshift, a brighter supernova means the supernova is closer. Closer means the supernova has a lower lookback time. Less lookback time for the same amount of expansion (redshift) during that time means a faster expansion. (Same expansion in less time.) Thus, $\boxed{\text{de Vaucoulers}}$ would have measured the brighter supernova.

2.
- The expansion of the Universe / cosmological redshift
- The Cosmic Microwave Background
- The ratios of light elements match what's calculated for what was produced early in the Universe

3. **(a)**

$$
\begin{aligned}
z &= \frac{d}{c\,t_H} \\
d &= c\,z\,t_H \\
d &= \left(\frac{1\,\text{lyr}}{yr}\right)(0.158)\,(13.8\times10^9\,\text{yr}) \\
d &= 2.18\times10^9\,\text{lyr}\left(\frac{1\,\text{pc}}{3.26\,\text{lyr}}\right) = \boxed{6.69\times10^8\,\text{pc} = 669\,\text{Mpc}}
\end{aligned}
$$

(b) The Universe is 0.158 times bigger (that's just the redshift!). Another way of saying the same thing is to say that the size of the Universe now is 1.158 times the size of the Universe when the light was emitted.

(c) We calculated the distance in light-years above, so it's easy to divide by the speed of light (1 light-year per year) to get $\boxed{\text{2.18 billion years}}$.

(d) It's a Uniform expansion, so t_H needs to be the same everywhere. $\boxed{\text{13.8 Gyr}}$.

(e) If the expansion rate has been constant, the current age of the Universe is 13.8 billion years. 2.18 billion years ago, the assumed-constant-expansion age of the Universe was 13.8-2.18 = $\boxed{11.6\,\text{Gyr}}$.

4. **(a)** Many of you made the mistake of saying that if t_H is the same, the age must be the same. This is not right. t_H tells you the age of the Universe *if the expansion rate of the Universe has been constant*. Since it hasn't been, t_H won't necessarily be equal to the age of the Universe.

There is more Dark Energy in the other Universe than in ours, so the other Universe should be experiencing more acceleration. However, the current expansion rate of both Universes (given by t_H) is the same. The other Universe was accelerating *more*, and thus was even slower in the past as compared to ours (so that the difference, and thus the acceleration, would be larger). Same rate now, slower rate in the past, thus it would have taken longer to get where it is right now. The other Universe would be $\boxed{\text{older}}$.

(b) Here are some:

- With no dark matter, there would be less gravity in galaxies to hold the stars in. Thus, the stars would need to moving slower if they are to stay in the galaxies. Orbital speeds of stars in galaxies should be lower.
- With no dark matter, there is only luminous matter. Thus, orbital speeds of stars in galaxies would match those predicted from looking at the luminous matter. (Note: the rotation curves would not be Keplerian! That requires all the mass at the center. While the dark matter is more spread out than the luminous matter, even the luminous matter is not all concentrated at the center.)

- The random velocities of galaxies in clusters of galaxies would be lower. With no dark matter to hold them in, they would fly apart if they had the velocities they have in our Universe.
- The acceleration of the expansion rate would be faster. (Note: the expansion rate itself would not be faster, because we were given that t_H is the same!)
- Gravitational lensing of the bullet cluster would show that most of the mass is where most of the luminous mass is. In that cluster, most of the mass is in the hot gas that got stuck in between the two clusters as they passed through each other. Gravitational lensing would show that most of the mass was right there, in contrast to what we see in our own Universe. (Note: many of you said gravitational lensing wouldn't happen, which is wrong. *Any* mass can cause gravitational lensing. In our Universe, gravitational lensing by a galaxy is dominated by dark matter simply because the galaxy is mostly dark matter, not because dark matter has an particular special influence on gravitational lensing.)