Astronomy 102, Review Exam 2

Solutions

1. **(a)** None. B stars live at most tens of millions of years. Indeed, an A star like Vega will only live several hundred millions of years. As such, any O and B stars that formed with the cluster are long since gone.

(b) All of 'em. Solar mass stars live 10 billion years, so they're just a tenth of their way into their life after 1 billion years.

(c) All of 'em. Red dwarfs are lower mass than the Sun, and live even longer. One billion years is much shorter than the lifetime of any red dwarf stars.

(d) The number of red giants should *increase* with time. Red giants are the stars that are just now in the last 10% of their life. As time goes by, lower and lower mass stars will be reaching the end of their lives. Additionally, when stars form, the lower the mass of the star, the more common that type of star is. Therefore, as time goes by, the number of stars that is just now reaching the last 10% of their life increases. Yes, the stars that had been giants will supernova, or turn into white dwarves, but the rate at which stars are becoming red giants goes up as the lifetime of lower and lower mass stars is reached.

2. **(a)** First of all, because the two stars are a binary system, they are the same distance away from us. Therefore, since $B_A = 9200\,B_B$, we also know that $L_A = 9200\,L_B$.

We also have the temperatures of the two stars, so we can do:

$$\begin{aligned} L_A &= 9200\,L_B \\ (4\pi\,{R_A}^2)\,\sigma\,{T_A}^4 &= 9200\,(4\pi\,{R_B}^2)\,\sigma\,{T_B}^4 \end{aligned}$$

Divide out the stuff that shows up on both sides:

$${R_A}^2\,{T_A}^4 = 9200\,{R_B}^2\,{T_B}^4$$

What we're looking for is R_A/R_B, so isolate that on the left side:

$$\begin{aligned} \left(\frac{{R_A}^2}{{R_B}^2}\right) &= 9200\left(\frac{{T_B}^4}{{T_A}^4}\right) \\ \frac{R_A}{R_B} &= \sqrt{9200}\left(\frac{T_B}{T_A}\right)^2 \\ \frac{R_A}{R_B} &= \sqrt{9200}\left(\frac{30,000}{10,000}\right)^2 \\ \frac{R_A}{R_B} &= 860 \end{aligned}$$

Sirius B is tiny for a star.... (Sirius A is a main sequence star, and Sirius B is white dwarf.)

(b) Here, we only have to worry about this:

$$B = \frac{L}{4\pi\,d^2}$$

Since we want a ratio of brightnesses, we can just divide the two of them over each other, and then substitute in the right side of the equation.

$$\frac{B_S}{B_\odot} = \frac{\frac{L_S}{4\pi\,{d_S}^2}}{\frac{L_\odot}{4\pi\,{d_\odot}^2}}$$

$$\frac{B_S}{B_\odot} = \left(\frac{L_S}{L_\odot}\right)\left(\frac{d_\odot}{d_S}\right)^2$$

We have numbers we can put in for the distances. We also have numbers we can put in for the luminosities, since we can look up $L_\odot$ on the front of the test, but it's easier to do it in units of solar luminosity; as long as they're the same as each other, they will cancel out.

$$\frac{B_S}{B_\odot} = \left(\frac{25\,L_\odot}{1\,L_\odot}\right)\left(\frac{1\,\mathrm{AU}}{8.6\,\mathrm{lyr}}\right)^2$$

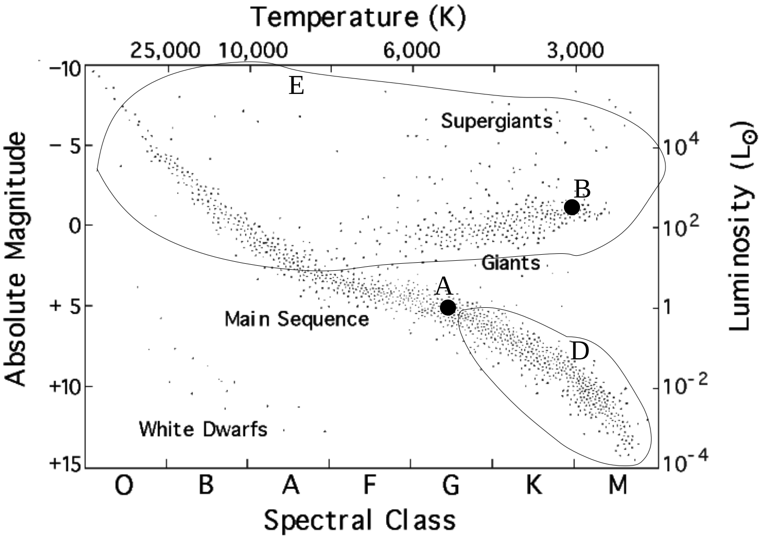
Well, OK, the luminosity units divide out, but the distance units don't. We have to put in some conversion factors. Notice that we have to put in the right number of conversion factors— the AU and lyr units are *squared.* Basically, what we're doing is converting both AU and lyr to pc, so that we'll have distances in consistent units.

$$\frac{B_S}{B_\odot} = 25\left(\frac{1\,\mathrm{AU}}{8.6\,\mathrm{lyr}}\right)^2\left(\frac{1\,\mathrm{pc}}{206265\,\mathrm{AU}}\right)^2\left(\frac{3.26\,\mathrm{lyr}}{1\,\mathrm{pc}}\right)^2$$

$$\frac{B_S}{B_\odot} = 8.4\times10^{-11}$$

Yup, the Sun is a whole lot brighter. . . as we knew it was!

3.



(b) You could just do it by remembering that the star is a red giant, there for much more luminous, and stick it somewhere in the giants branch at around 3,000K. Or, you could calculate the luminosity:

$$\frac{L_{\mathrm{RG}}}{L_\odot} = \frac{4\pi\,(100\,R)^2\,\sigma\,(3000\,K)^4}{4\pi\,(R)^2\,\sigma\,(6000\,K)^4}$$

$$\frac{L_{\mathrm{RG}}}{L_\odot} = (100)^2\left(\frac{1}{2}\right)^4$$

$$L_{\mathrm{RG}} = 600\,L_\odot$$

(Really the math gives you a ratio of 625, but this calculation only has one significant figure.) Interpolate the vertical axis as best as you can to put the point somewhere reasonable. (It's a bit tricky because it's a logarithmic scale, but as long as it's above 10^2 and below where 10^3 is, you'd get full credit.)

(f) Color going from bluer to redder, and therefore also a sequence of decreasing surface temperature. It is *not* a sequence of decreasing luminosity or brightness! (It is decreasing luminosity and increasing lifespan for *main-sequence stars only.*)

4. (a)

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{orig}}}{\lambda_{\text{orig}}}$$

$$z = \frac{6665.5\text{Å} - 6562.8\text{AA}}{6562.8\text{AA}}$$

Note: in the subtraction in the numerator (which yields 102.7 angstroms), we reduce the number of sig figs we have from what looks like 5 down to 3 or 4.

$$z = 0.0156$$

(b) The galaxy is moving away from us, because the wavelength of the emission line is shifted to the red (to higher wavelengths). In the (not entirely correct) interpretation that this z is due to a Doppler Shift, we would say that it's speed of recession is:

$$z = \frac{v}{c}$$

$$v = c\,z = (2.998 \times 10^5\,\text{km/s})\,(0.0156)$$

$$v = 4690\,\text{km/s}$$

(c) Do the same basic thing to find the Doppler Shift we're ascribing to the NW gas:

$$\frac{v}{c} = \frac{\lambda_{\text{obs}} - \lambda_{\text{orig}}}{\lambda_{\text{obs}}}$$

$$v = (2.998 \times 10^5)\left(\frac{6664.8\text{Å} - 6562.8\text{Å}}{6562.8\text{Å}}\right)$$

$$v = 4660\,\text{km/s}$$

Because this speed is lower than what we had in (b) above, it means that, *compared to the nucleus*, this gas is coming $\boxed{\textit{towards}\text{ us at a speed of 30 km/s}}$. (There is only one sig fig left in that final number there!)

(d) Do the same basic thing to find the Doppler Shift we're ascribing to the SE gas:

$$\frac{v}{c} = \frac{\lambda_{\text{obs}} - \lambda_{\text{orig}}}{\lambda_{\text{obs}}}$$

$$v = (2.998 \times 10^5)\left(\frac{6667.4\text{Å} - 6562.8\text{Å}}{6562.8\text{Å}}\right)$$

$$v = 4780\,\text{km/s}$$

Because this speed is higher than what we had in (b) above, it means that, *compared to the nucleus*, this gas is going $\boxed{\textit{away from}\text{ us at a speed of 90 km/s}}$. (There is only one sig fig left in that final number there!)