November 25, 2006

A102 Fall 2006 Review Exam 4 Solutions

Note: my answers are more verbose than yours will generally need to be. I am looking for you to explain it well enough that I know you fully understand the concepts. Here, I'm trying to explain it well enough so that you can understand the answers to the questions.

1. A dimmer peak brightness means that the supernova is farther away. (Because the luminosity is always the same, in the absence of dust or other problems, more distant supernovae will appear dimmer.)

If the supernova is farther away, then it took the light longer to reach us than it would have been if it had been closer. This means that a *dimmer supernova equates to a longer lookback time.*

The redshift is the amount of expansion during the lookback time. A fixed amount of expansion in a longer time would be a *lower* expansion rate. (It took longer to expand that amount.)

Thus, a *dimmer supernova would indicate a lower expansion rate.* This may seem counter-intuitive if you try to jump straight to the interpretation. Carefully think through the steps described above.

2. **(a)** Five billion years from now, the Universe will be five billion years older. The Hubble Time is the age of the Universe *if the expansion rate is constant.* Since we're assuming that, five billion years from now the Hubble Time would be 13.8 billion years plus 5 billion years, or $\boxed{\text{18.8 billion years}}$.

(b) The Universe's expansion is *accelerating.* Thus, five billion years from now, people will measure a *faster* expansion rate. The Hubble Time is how long it took the Universe to reach the current size *assuming the expansion rate has been constant.* Five billion years from now, people will make that assumption with a *faster* expansion rate than we have, so they will measure a Hubble Time that is $\boxed{\text{less than 18.8 billion years}}$.

(Why not less than 13.8 billion years? Because they will be calculating how long it took for the Universe to reach a size that is bigger than what we calculate for today.)

3. **(a)**

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm orig}}{\lambda_{\rm orig}} = \frac{6694.0\,\text{Å} - 6562.8\,\text{Å}}{6962.8\,\text{Å}}$$

$$\boxed{z = 0.01999}$$

(Do you understand why there are four, not five, significant figures? I would, however, give full credit if you just said "0.02" or "0.020" or "0.0200".)

(b) The expanding Universe equation:

$$z = \frac{d}{c\,t_H}$$

$$d = z\,c\,t_H = (0.01999)\,(1\,\text{lyr}\,\text{yr}^{-1})\,(13.8\times 10^9\,\text{yr})$$

$$d = 2.76\times 10^8\,\text{lyr}\left(\frac{1\,\text{pc}}{3.26\,\text{lyr}}\right) = 8.46\times 10^7\,\text{pc} = \boxed{84.6\,\text{Mpc}}$$

(c) 276 million years (see the distance in light-years in (b).

(d) We have $L_C = 1,200\,L_\odot$ and $L_V = 130\,L_\odot$. We also know the distances to this star and to Vega. As such, it's pretty simple to directly use the Brightness equation:

$$\frac{B_C}{B_V} = \frac{\frac{L_C}{4\pi\,{d_C}^2}}{\frac{L_V}{4\pi\,{d_V}^2}}$$

$$\frac{B_C}{B_V} = \left(\frac{L_C}{L_V}\right)\left(\frac{d_V}{d_C}\right)^2$$

$$\frac{B_C}{B_V} = \left(\frac{1,200\,L_\odot}{130\,L_\odot}\right)\left(\frac{7.76\,\mathrm{pc}}{84.6\times10^6\,\mathrm{pc}}\right)^2$$

$$\boxed{\frac{B_C}{B_V} = 7.77\times10^{-14}}$$

(e) Nope! Sorry. This is why we can't easily use Cepheid Variables to measure distances to galaxies that are 85 Mpc away.

4. (a) It means that the Universe is homogeneous and isotropic.

Homogeneous means "the same everywhere." If you choose a region of a billion light-years around the Milky Way, and a region of a billion light-years centered around some very distant galaxy, you should find pretty much the same kinds of things— galaxy clusters, dark matter, and dark energy in the same densities and distributed in the same sorts of ways.

Isotropic means "the same in all direction". If you look very far in the direction of the constellation Virgo, and very far in the direction of the constellation Pegasus, you should see the same sorts of galaxies at the same sorts of redshifts.

Here are some pieces of evidence that the Cosmological Principle is valid:

- Large scale galaxy surveys indeed show that the distribution of galaxies is the same at different places in the Universe, and same in different directions.
- Cosmic Microwave Background light coming from all different directions has the same temperature.
- Expanding universe equations developed from the assumptions of an isotropic and homogeneous Universe give a good description of the relationship between redshift and lookback time.
- The expansion rate (and history of the expansion rate) is the same if you look at galaxies at different distances in all different directions.

(b) We sure seem to be in a special place! This would seem to be a violation of the Cosmological Principle. However, the Cosmological Principle only applies on the largest scales. Obviously there are local differences; some places are inside galaxies (which will have a high Dark and Normal matter density), other places arena. But, if you average over a large enough box (say a billion light-years across), the average density and average sorts of contents you have in that box, and in another box far away, will be the same.

5. (a) Dark Energy is what is making the Universe's expansion accelerate. Without that, all the matter (normal and dark) would be exerting an attractive gravitational pull on itself, causing the Universe's expansion to decelerate.

(b) The ratio of Dark Matter to normal matter in this hypothetical Universe is a lot higher than it is in our Universe. As such, within a galaxy, there's *more* gravitational pull holding stars in the galaxy than there is in our real Universe.

If there's more gravitational pull, the stars will tend to collapse towards the center. To avoid that, they'd have to be moving even faster than they do in our galaxy. That leads to the first conclusion: galaxies would have faster rotation speeds.

If you also consider that dark matter is spread out more than normal matter, it makes a bigger and bigger difference compared to normal matter as you get farther and farther away from the galaxy. Thus, near the center, where normal and dark matter both matter, instead of a flat, you'd see a *rising* rotation curve for galaxies in this hypothetical dark-matter-rich Universe.

(c) It wouldn't change at all. The amount of dark matter in our Solar System is *tiny* compared to the amount of normal matter in our Solar System. The mass of the Sun (which is what governs planetary rotation speeds) is so much more that the mass of the dark matter that even if there were three times as much dark matter, it would still be irrelevant within the Solar System.