A102 Fall 2006 Review Final Solutions

1. The first thing to recognize is that 12 half-lives have elapsed. The total initial amount of Potassium-40 was 4096 the current amount, since everything that has decayed has decayed to Argon-40. If 12 half-lives have elapsed,then $12 \times 1.26\,\text{Gyr} = 15.1$ billion years have elapsed. Since our solar system is in the Universe, the Universe must be at least that old. (Or, would would conclude that if we really observed that ratio.) This is at odds with what we've observed from the Big Bang theory, so it would make us very nervous that we haven't calculated the age of the Universe right.

Fortunately for those of us in the real Universe, our Solar System is only 4.6 billion years old, which fits comfortably within the 13.7 billion years we calculate for the Universe.

2. **(a)** If 3.8×10^{26} J of energy are produced each second, then, by $E = mc^2$, each second we must be converting to energy this much mass:

$$m = \frac{E}{c^2} = \frac{3.8 \times 10^{26}\,\text{J}}{(3.0 \times 10^8\,\text{m}\,\text{s}^{-1})^2} = 4.222 \times 10^9\,\text{kg}$$

So $\boxed{4.2 \times 10^9 \text{ kg}}$ of mass is converted to energy each second.

(b) The efficiency is 0.007, so the amount of fuel that needs to be used up to convert 4.2×10^9 kg of mass to Energy is:

$$\frac{4.2 \times 10^9\,\text{kg}}{0.007} = \boxed{6.032 \times 10^{11}\,\text{kg}}$$

(c) Each fusion reaction uses up 4 H nuclei, so the number reactions that must happen in order to use up 6.0×10^{11} kg of Hydrogen is:

$$\frac{6.032 \times 10^{11}\,\text{kg}}{(4)(1.67 \times 10^{-27}\,\text{kg})} = \boxed{9.030 \times 10^{37}}$$

That's a lot of fusion reactions...!

(d) Each fusion reaction produces 2 neutrinos (see the equation sheet), so the number of neutrinos produced per second is $2 \times 9.030 \times 10^{37} = \boxed{1.8 \times 10^{38}}$.

(e)

$$4\pi\,(1.496 \times 10^{11}\,\text{m})^2 = 2.812 \times 10^{23}\,\text{m}^2$$

The surface area is $\boxed{2.8 \times 10^{23}\,\text{m}^2}$

(f) One cm^2 is $(0.01\,\text{m})^2$, or $10^{-4}\,\text{m}^2$. Your thumb is this fraction of the area that the neutrinos are spread overq:

$$\frac{10^{-4}}{2.812 \times 10^{23}} = 3.556 \times 10^{-28}$$

The fraction is $\boxed{3.6 \times 10^{-28}}$.

(g) The number of neutrinos going through your thumb each second is the number emitted by the Sun each second, times the fraction of the area that the neutrinos are spread over represented by your thumb:

$$(1.806 \times 10^{38})\,(3.556 \times 10^{-28}) = 6.42 \times 10^{10}$$

There are *60 billion neutrinos* going through your thumb every second!!!!! Yipers.

NOTE: I think I've done something wrong. The answer should have come out to 6 million, not 60 billion, but I don't see where I went wrong. . . .

3. **(a)** The one on the left is the one that formed recently, the one on the right is the one that formed a long time ago.

   (b) The main-sequence turnoff. The main-sequence is the group of stars in a diagonal line from upper-left to lower-right. On the main-sequence, more massive, short-lived stars are both hotter and more luminous (to the upper-left in the diagram). As time goes by, stars of lower and lower mass will reach the ends of their lives and move off of the main sequence. The cluster on the right shows a clear, lowish-temperature main sequence turnoff, whereas the cluster on the left only barely has a turnoff at relatively high temperature and luminosity.

   (c) Calculations of where stars of various masses and ages should lie on the H-R diagram, produced from the theory of stellar evolution.

   (d) The oldest globular clusters (clusters like the one whose H-R diagram is on the right) show a main-sequence turnoff that matches an age of 12–13 billion years, so the Universe must be at least that old.

4. **(a)** If the expansion rate has always been constant, then the rate at which a given galaxy is getting farther away has always been what it is right now. If we find the distance to that galaxy, and divide it by the expansion rate (in some sort of speed-like unit), we'll get the amount of time it took for the galaxy to get to that distance from right on top of us to its current distance.

   (b)

$$z = \frac{d}{c\,t_H}$$

$$t_H = \frac{d}{c\,z} = \left(\frac{120 \times 10^6\,\mathrm{pc}}{(1\,\mathrm{lyr/yr})(0.023)}\right)\left(\frac{3.26\,\mathrm{lyr}}{\mathrm{pc}}\right)$$

$$\boxed{t_H = 17\,\text{billion years}}$$

   (c) At the current expansion rate, it would have taken 17 billion years for a given galaxy to get from right here to its current distance. However, it only took 15 billion years. As such, it got out there faster than it would have if the expansion rate had always been constant, and thus the expansion rate must have been faster in the past. That's a $\boxed{\text{decelerating}}$ Universe.

5. **(a)**

$$d = \frac{1}{p} \quad \text{, so}$$

$$\frac{d_A}{d_B} = \frac{p_B}{p_A} = \frac{.05}{.1} = \boxed{0.5}$$

   (b) If the stars have the same diameter, then only temperature makes a difference in Luminosity. Luminosity goes as temperature to the fourth, so:

$$\frac{L_A}{L_B} = \left(\frac{T_A}{T_B}\right)^4 = \left(\frac{2}{1}\right)^4 = \boxed{16}$$

   (c) If they were unobscured, the ratio of brightnesses would be:

$$\frac{B_{A*}}{B_B} = \frac{\frac{L_A}{4\pi\,{d_A}^2}}{\frac{L_B}{4\pi\,{d_B}^2}}$$

$$\frac{B_{A*}}{B_B} = \left(\frac{L_A}{L_B}\right)\left(\frac{d_B}{d_A}\right)^2$$

$$\frac{B_{A*}}{B_B} = (16)(\frac{1}{0.5})^2 = 64$$

   In fact, star A is only one eight this bright, so the $\boxed{\text{dust is dimming by a factor of 8}}$.

6. First of all, the planetary nebula phase is a *very* brief phase of a star's existence. Low mass stars can live many billions of years, but a planetary nebula only lasts for a few ten thousand years. The chance of catching a star right at the moment when it is putting out a planetary nebula is very low, even if stars have been formed at all different times.

Second of all, stars less massive than about 0.8 times the mass of the Sun have lifetimes that are longer than the age of the Universe. Thus, *none* of the lowest-mass stars have yet thrown out a planetary nebula, which only serves to increase the number of low-mass stars without contributing at all to the number of planetary nebulae.

7. **(a)** It won't change. As space expands, there will be more volume to spread matter over, so the density of each will go down... but they will go down at the same rate, so the ratio of normal to dark matter will stay the same.

(b) It will go up. The density of dark matter goes down as the volume increases, because the dark matter is getting more and more spread out. However, dark energy has a constant density.

(c) The ratio of normal matter to dark energy right now is (5%/70%), or 0.0071. When the CMB was emitted, the dark energy density was the same. However, because normal matter was squeezed into a volume that was 10^9 times larger, the density of normal matter was 10^9 times higher. As such, the ratio of normal matter to dark energy was 10^9 times higher, or 70 million. Dark energy wasn't too important back then....

(d) Two things were different when the Universe was much smaller. First, it was much denser. Second, it was much hotter. (For instance, all of the photons hadn't been redshifted by a factor of 1000, and thus had shorter wavelengths and more energy.) When it was hot enough, the Universe was a plasma, with photons and electrons separate. The density and nature of the Universe made it *opaque*; light couldn't travel freely very far. As it expanded, eventually it got cool enough and low density enough to thin out and become transparent. That transition was the CMB. The Universe is very transparent right now. A change of only 2% in the density and in the wavelengths of light wouldn't be nearly enough to heat up and compress the material in the Universe to make it opaque. It had to have happened way the heck back when, when the Universe was a lot smaller than it is right now.