Astro 102 Review Problem Set #1
Solutions

1. The total *amount* here is mass: it's a mass of 10^{38} kg. The rate at which mass is being eaten is 2×10^{30} kg per year. So, we can find the time it will take by dividing the amount by the rate:

$$t = \frac{1 \times 10^{38}\,\text{kg}}{2 \times 10^{30}\,\frac{\text{kg}}{\text{yr}}} = 5 \times 10^{7}\,\text{years} = 50\text{ million years}$$

2. **(a)** This is an example of a "proportionality" problem. What fraction of one thing is another thing? How many times one thing is another thing? In this case, the question is, what fraction of the Sun's mass is the amount of mass eaten by the quasar each year?

$$f = \frac{\text{Amount eaten per year}}{\text{Sun's mass}} = \frac{2.0 \times 10^{30}\,\text{kg}}{2.0 \times 10^{30}\,\text{kg}} = 1$$

In other words, this black hole at the core of this quasar is eating one solar mass per year.

(b) If it is eating one solar mass per year for 50 million years, it will eat 50 million solar masses! This is a simple example of a rate problem:

$$m = \left(\frac{1\,M_\odot}{\text{year}}\right)(5 \times 10^{7}\,\text{years}) = 5 \times 10^{7}\,\text{M}_\odot$$

The $\odot$ symbol means "the Sun", so a M with a $\odot$ subscript is the mass of the Sun, or the solar mass.

3. The room is about 10m wide (very roughly). How long does it take light to cross this room?

$$t = \frac{10\,\text{m}}{3.00 \times 10^{8}\,\frac{\text{m}}{\text{s}}}$$
$$= 3.33 \times 10^{-8}\,\text{s}$$

Since light goes one light-second in one second, that means that the room is 3.33×10^{-8} light-seconds wide. **Note:** If you get an answer to a question like this which is comparable to or larger than a second, you should sit back and scratch your head. You know that light takes *almost no* time (given how humans reckon time) to cross something as small as this room. If the number you get out of your calculation doesn't match that foreknowledge, then either your foreknowledge is mistaken, or you've made an error in the problem. Always make sure that your answers make sense!

4. Although the rate is changing as the number of atoms decrease, let's make the approximation that the rate is constant. You know that half what you start with will be gone in 5,700 years, so a decent approximation at the rate of decay is:

$$\frac{0.50 \times 10^{12}\,\text{decays}}{5,700\,\text{years}}$$
$$= 8.772 \times 10^{7}\,\frac{\text{decays}}{\text{year}}$$

(Note that I've kept two more significant figures than I have, because this number will be used in subsequent calculations.)

Now I need to convert this to a rate per second:

$$\left(8.772 \times 10^{7}\,\frac{\text{decays}}{\text{year}}\right)\left(\frac{1\,\text{year}}{3.156 \times 10^{7}\,\text{s}}\right) = 2.779\,\frac{\text{decays}}{\text{s}}$$

Multiply the rate times the time to figure out the total amount.

$$\left(2.779\,\frac{\text{decays}}{\text{s}}\right)(10\,\text{s})$$

$$\boxed{= 28\ \text{decays}}.$$

5. **(a)** The Solar System is 4.6 billion years old. Reading the graph, the fraction of Potassium-40 left is about 0.07. this means that the fraction that has been converted to Argon-40 is (1-0.07), or about 0.93. Thus, the ratio is:

$$\frac{(0.07)(\text{amount Potassium} - 40\ \text{we started with})}{(0.93)(\text{amount Potassium} - 40\ \text{we started with})}$$

$$\boxed{= 0.08}$$

I only give the answer to one sig fig, because that's about as well as I think I've read the graph.

(b) We want the amount of Argon-40 to be three times the amount of Potassium-40. Since Potassium is converted to Argon, if 0.25 of the original Potassium-40 is left, then 0.75 has been converted to Argon; that gives us a 1/3 ratio. Looking at a the plot, that happens at about $\boxed{\text{2.5 billion years}}$.

(c) Now the situation is opposite, so we want the amount of Potassium-40 left to be 0.75. Reading the graph, that happens at about $\boxed{\text{0.5 billion years}}$.

6. The distance travelled over the next 100 years is:

$$
\begin{aligned}
&\left(1,400\,\frac{\text{km}}{\text{s}}\right)(100\,\text{yr})\left(\frac{3.156\times 10^{7}\,\text{s}}{1\,\text{yr}}\right)\\
&= 4.418\times 10^{12}\,\text{km}\left(\frac{1\,\text{AU}}{1.5\times 10^{8}\,\text{km}}\right)\left(\frac{1\,\text{pc}}{206,265\,\text{AU}}\right)\left(\frac{3.26\,\text{lyr}}{1\,\text{pc}}\right)\\
&= 0.47\,\text{lyr}
\end{aligned}
$$

Notice in the first line I multiplied a rate by a time, but also had to convert the time units to be consistent (leaving me with km as the only unit). Notice in the second line that I had a distance in km, but converted it to light-years. The distance that the galaxies travel is equal to the difference in distances between 100 years from now and now. The ration of that distance to the current distance is:

$$
\frac{0.47\,\text{lyr}}{65\times 10^{6}\,\text{lyr}} = 7\times 10^{-9}
$$

That is, a very tiny fraction. In a hundred years, the Universe will expand by only about a millionth of a percent.