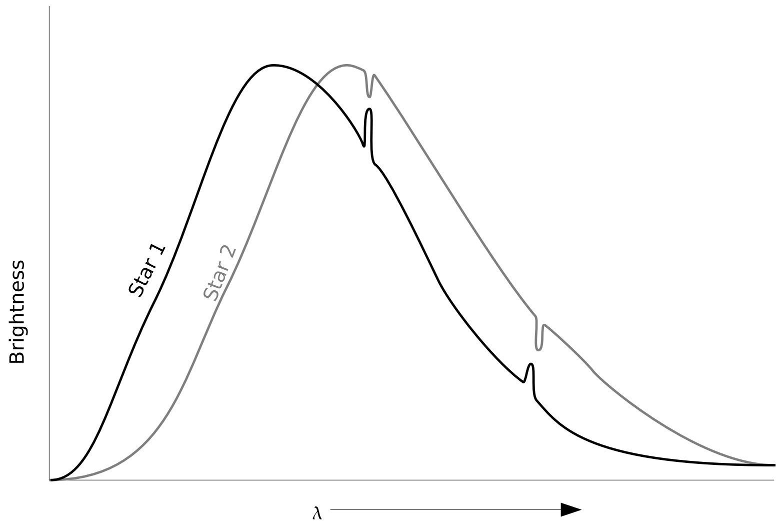
Astro 102 Review Problem Set #2
Solutions
Revision 3 — fixed an error in 5c; completed 5c; deleted spurious prob. 13

1.



2. **(a)** We don't know the radius or temperature, so we can't get the luminosity that way. However, we do know how bright it is compared to the Sun, and how far away it is. . . and what we want is the luminosity of Betelgeuse relative to the Sun. So we can use:

$$B = \frac{L}{4\pi\, d^2}$$

Solve this for Luminosity:

$$L = B\, 4\pi\, d^2$$

Make one of these equations for Betelgeuse, and one for the Sun. Divide the two equations:

$$\frac{L_B}{L_\odot} = \frac{B_B\, 4\pi\, {d_B}^2}{B_\odot\, 4\pi\, {d_\odot}^2}$$

$$\frac{L_B}{L_\odot} = \left(\frac{B_B}{B_\odot}\right)\left(\frac{d_B}{d_\odot}\right)^2$$

$$\frac{L_B}{L_\odot} = (1.4 \times 10^{-10})\left(\frac{130\,\mathrm{pc}}{1\,\mathrm{AU}}\right)^2$$

Notice that we have to do a unit conversion; put that in:

$$\frac{L_B}{L_\odot} = (1.4 \times 10^{-10}) \left(\frac{130\,\mathrm{pc}}{1\,\mathrm{AU}}\right)^2 \left(\frac{206,265\,\mathrm{AU}}{1\,\mathrm{pc}}\right)^2$$

$$\frac{L_B}{L_\odot} = 1.0 \times 10^5$$

So we have that Beteleguse is 100,000 times as luminous as the Sun (to two sig figs).

(b) We now know luminosity and temperature compared to the Sun, and we can look up the Sun's radius on the front page, so it's a matter of using this equation:

$$L = (4\pi\, R^2)\, \sigma\, T^4$$

Just for variety, let's do this a slightly different way. The "divide the equations" trick would work here too, but we can also start with an equality we know, and work from there. We know from part (a) that :

$$L_B = 1.007 \times 10^5\, L_\odot$$

$$(4\pi\, {R_B}^2)\, \sigma\, {T_B}^4 = 1.007 \times 10^5\, (4\pi\, {R_\odot}^2)\, \sigma\, {T_\odot}^4$$

The 4π and σ terms appear on both sides, so we can divide them away. Also divide out the ${T_B}^4$ term to get:

$${R_B}^2 = 1.007 \times 10^5\, {R_\odot}^2 \left(\frac{T_\odot}{T_B}\right)^4$$

$$R_B = \sqrt{1.007 \times 10^5}\, R_\odot \left(\frac{T_\odot}{T_B}\right)^2$$

Now we can put in numbers. Note that we have the radius of the Sun from the front of the test as 6.97×10^8 m, which is the same as 6.97×10^5 km.

$$R_B = \sqrt{1.007 \times 10^5}\, (6.97 \times 10^5\,\mathrm{km}) \left(\frac{5,800\,\mathrm{K}}{3,000\,\mathrm{K}}\right)^2$$

$$R_B = 8.3 \times 10^8\,\mathrm{km}$$

(c) 1 AU is 1.5×10^{11} m, or 1.5×10^8 km. Thus, the radius of Betelgeuse is about 5.5 times the radius of the Earth's orbit— or, a bit bigger than the radius of Jupiter's orbit!

3. The simplest reason is the simple fact that the wavelength of each and every photon emitted by the source is redshifted when detected by the observer. Therefore, each photon detected by the observer has a longer wavelength, and thus a *lower* energy than it would have had if the source had been at rest with respect to the observer. If the photon emission rate is the same, then the observer is seeing photons at the same rate, but with a lower average energy per photon. Thus, the observer sees a lower energy rate overall.

(It turns out there is a second reason, that has to do with Special Relativity (SR). In SR, we learn that moving clocks run slow. Thus, the photon rate (which is a sort of "clock") is actually a bit *lower* than it would have been if the source were at rest. However, these SR considerations aren't anything we've dealth with in this class, and I don't expect you to know about them or realize that they might be relevant here.)

(Katie, you and only you might know about the third "aberration" effect, becuase I went over it at one point in Astro 311...it was when I was talking about relativistic beaming.)

4. The most negative velocity you will get is when the gas is coming towards you and is therefore blueshifted to the lowest observed wavelength:

$$\begin{aligned}
\frac{v_{\rm min}}{c} &= \frac{6560\,\text{Å} - 6563\,\text{Å}}{6563\,\text{Å}} \\
v_{\rm min} &= (-0.000457)(3.00 \times 10^8\,\frac{\rm m}{\rm s} \\
v_{\rm min} &= -140,000\,\frac{\rm m}{\rm s} \\
v_{\rm min} &= -140\,\text{km/s}
\end{aligned}$$

Similarly, the highest velocity you will get is when the gas is receeding and therefore redshifted to the lowest observed wavelength:

$$\begin{aligned}
\frac{v_{\rm min}}{c} &= \frac{6566\,\text{Å} - 6563\,\text{Å}}{6563\,\text{Å}} \\
v_{\rm min} &= (+0.000457)(3.00 \times 10^8\,\frac{\rm m}{\rm s}) \\
v_{\rm min} &= 140,000\,\frac{\rm m}{\rm s} \\
v_{\rm min} &= 140\,\text{km/s}
\end{aligned}$$

Thus, the range is $\boxed{\text{-140 to 140 km/s}}$, or equivalently everything between 140 km/s approaching and 140 km/s receeding.

Note that the "everything between" is important! In the past, when I had assigned this problem as a homework problem, a lot of people said "-140 or 140 km/s" or "either -140 or 140 km/s". This is not right! In that case, you would see a split line, with two very narrow lines at 6560Å and 6563Å. To get a *broadened* line, you need a continuous range of velocities.

A note on significant figures: Really the answer to this problem should only have *one* significant figure! Why only one, when it looks like the initial numbers are given to four sig figs? Go back and look at the Math Review again. Notice that you are subtracting two numbers, 6560-6563. That subtraction is only significant to the *ones* place, and that's the point at which your number of significant figures gets reduced to one. In practice, astronomers can often measure wavelengths to within a tenth of an angstrom or better, depending on the resolution of the spectrometer they are using.

5.

(a) Remember that there are *two* photons, but also *two* particles. The energy in the mass of these two particles is converted into the energy of the photons:

$$2\,E = 2\,m\,c^2$$
$$E = m\,c^2$$
$$E = (9.1 \times 10^{-31}\,\text{kg})\,(3.0 \times 10^{8}\,\frac{\text{m}}{\text{s}})^2$$
$$\boxed{E = 8.2 \times 10^{-14}\,\text{J}}$$

(b)

$$E = h\,f \qquad f\,\lambda = c$$
$$f = \frac{E}{h} \qquad \lambda = \frac{c}{f}$$

Put these two together:

$$\lambda = \frac{h\,c}{E}$$
$$\lambda = \frac{(6.626 \times 10^{-34}\,\text{J s})\,(3.0 \times 10^{8}\,\text{m/s})}{8.19 \times 10^{-14}\,\text{J}}$$
$$\boxed{\lambda = 2.4 \times 10^{-12}\,\text{m}}$$

(c) Start by figuring out the energy of one blue photon, using the equation from the previous part:

$$E_{\text{blue}} = \frac{h\,c}{4500\,\text{Å}}$$

Make sure to get these into consistent units! 4500 Å$= 4500 \times 10^{-10}$ m.

$$E_{\text{blue}} = \frac{(6.626 \times 10^{-34}\,\text{J s})\,(3.0 \times 10^{8}\,\text{m/s})}{4500 \times 10^{-10}\,\text{m}}$$
$$E_{\text{blue}} = 4.42 \times 10^{-19}\,\text{J}$$

To figure out how many blue photons have the same energy as one of the gamma photons, just divide the two energies:

$$\frac{E_{\text{gamma}}}{E_{\text{blue}}} = \frac{8.2 \times 10^{-14}}{4.42 \times 10^{-19}} = 1.9 \times 10^5$$

It takes about 190 thousand blue photons to make up the energy of just one of the gammay ray photons that comes out of electron/positron annihilation.

(d) Note that the fact that it is blue light doesn't actually matter; all you want is luminosity, which is energy produced per second. So there's no need to worry about the frequency of blue light or any such.

$$L = \frac{E}{t} = \frac{2\,m\,c^2}{1\,\text{s}}$$

The 2 is because we have 1ng of electrons and 1ng of positrons, for 2ng total (where m is 1ng).

$$L = \frac{2\,(10^{-12}\,\text{kg})\,(3 \times 10^8\,\text{m/s})^2}{1\,\text{s}}$$
$$L = 180,000\,\text{W}$$

This is way the heck more than a lightbulb. Using tiny amounts of matter/antimatter gets you a lot of power. This shows two things; first, that mass is a very efficient way to store energy. Second, that all those science fiction writers who think that starships might use antimatter as fuel might be onto something.