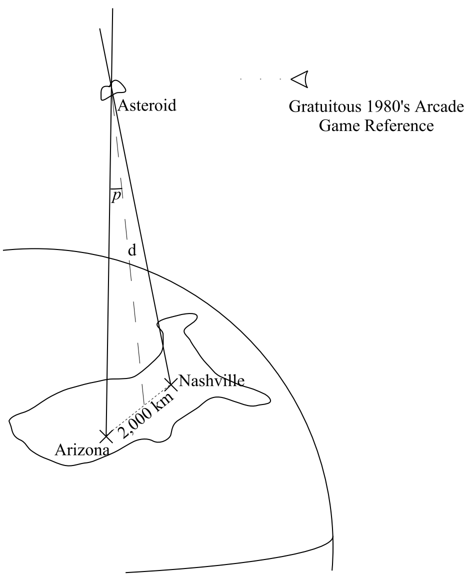
Astro 102 Review Problem Set #4
Solutions

1.

(a)



(b) The angle p is small ($2''$ is a small angle), so we can use the formula:

$$p = \frac{h}{d}$$

In this case, we know h (it's 1,000 km, or half of the baseline between Nashville and Arizona), and we want to find d, so we solve this equation:

$$d = \frac{h}{p}$$

To use this equation, though, we have to convert p from arcseconds to radians:

$$p = (2'') \times \frac{1 \text{ radian}}{206,265''} = 9.7 \times 10^{-6} \text{radians}$$

If we plug in this minimum angle we can measure, we'll get the maximum d we can measure. (Any d lower than that would give us a bigger angle, which would only be easier to measure with our precision.) This will give us d in km (the same units as we have h in).

$$d = \frac{1,000 \text{ km}}{9.7 \times 10^{-6}}$$

$$\boxed{d = 1 \times 10^{8} \text{ km}}$$

For purposes of comparison, convert this to AU:

$$d = 1.03 \times 10^8\,\text{km} \times \left(\frac{1\,\text{AU}}{1.496 \times 10^8\,\text{km}}\right) = \boxed{0.7\,\text{AU}}$$

(NOTE : if you used 2,000 km for the baseline, that's fine, and indeed, probably a better answer given the way the question was phrased. (Do you know why?))

(c) This is much larger than the distance from the Earth to the Moon, and is comparable to the distance from the Earth to the Sun.

(d) You will want it to be as high in the sky as it gets. If it's near rising or setting, the triangle will be "squished", giving you a smaller angle. Ideally, you want it to be just *past* its highest point in the sky as viewed from Nashville, and just *before* its highest point in the sky as viewed from Arizona, so that it's nicely positioned up between the two as drawn. Anywhere else, and you'll have to do some trig to take into account the "squished" triangle.

2. From the parallaxes, we have distances:

$$d_A = \frac{1}{0.10}\,\text{pc} = 10\,\text{pc}$$

$$d_B = \frac{1}{0.025}\,\text{pc} = 40\,\text{pc}$$

We are also given a brightness ratio:

$$B_B = 2\,B_A$$

What we're looking for is luminosities. B is farther, but B is also brighter— so we'd better come out with B being more luminous! We expect a ratio $L_A/L_B < 1$.

We have distances, and we have brightnesses. We have distances and brightnesses, and know how to relate all that to luminosities:

$$B = \frac{L}{4\pi\,d^2}$$

Specifically, we have:

$$B_A = \frac{L_A}{4\pi\,{d_A}^2} \qquad B_B = \frac{L_B}{4\pi\,{d_B}^2}$$

What we want is a luminosity ratio, so solve each of these for luminosity:

$$L_A = 4\pi\,{d_A}^2\,B_A \qquad L_B = 4\pi\,{d_B}^2\,B_B$$

To get the ratio, divide those two puppies:

$$\frac{L_A}{L_B} = \frac{4\pi\,{d_A}^2\,B_A}{4\pi\,{d_B}^2\,B_B}$$

$$\frac{L_A}{L_B} = \left(\frac{d_A}{d_B}\right)^2 \left(\frac{B_A}{B_B}\right)$$

$$\frac{L_A}{L_B} = \left(\frac{10\,\text{pc}}{40\,\text{pc}}\right)^2 \left(\frac{1}{2}\right)$$

$$\boxed{\frac{L_A}{L_B} = \frac{1}{32}}$$

This does come out with B more luminous, as expected.

3. The first Cepheid variable star's distance can be determined easily from the parallax:

$$d_{\text{C1}} = \frac{1}{p} = \frac{1}{0.022}$$

$$d_{\text{C1}} = 45.45\,\text{pc}$$

(I am keeping "too many" sig figs, since I'll use this number in subsequent calculations.)

The second Cepheid variable star is dimmer, but also farther. It has the same period, though, so it must have the *same* luminosity as the first Cepheid. (This is what makes Cepheids useful!)

$$B = \frac{L}{4\pi\, d^2}$$

$$L = 4\pi\, d^2\, B$$

$$L_{\text{C1}} = L_{\text{C2}}$$

$$4\pi\, {d_{\text{C1}}}^2\, B_{\text{C1}} = 4\pi\, {d_{\text{C2}}}^2\, B_{\text{C2}}$$

$$d_{\text{C2}} = d_{\text{C1}} \sqrt{\frac{B_{\text{C1}}}{B_{\text{C2}}}}$$

$$d_{\text{C2}} = 45.45\,\text{pc} \sqrt{\frac{3.4 \times 10^{-11}}{1.1 \times 10^{-21}}}$$

$$d_{\text{C2}} = 8.00 \times 10^6\,\text{pc} = 8.00\,\text{Mpc}$$

That is a reasonable distance for a "nearby" galaxy (galaxies tend to be a few Mpc apart).

The first Type Ia supernova is 8.00 Mpc away, of course, since it's in the same galaxy as the first Cepheid! We can do the same thing as above to figure out the ratio of distances between the two supernovae; remember that supernovae are standard candles.

$$L_{S1} = L_{S1}$$

$$4\pi\, {d_{\text{S1}}}^2\, B_{\text{S1}} = 4\pi\, {d_{\text{S2}}}^2\, B_{\text{S2}}$$

$$d_{\text{S2}} = d_{\text{S1}} \sqrt{\frac{B_{\text{S1}}}{B_{\text{S2}}}}$$

$$d_{\rm S2} = 8.00\,{\rm Mpc},\sqrt{\frac{1.5\times 10^{-15}}{1.3\times 10^{-19}}}$$

$$d_{\rm S2} = \boxed{860\,{\rm Mpc}} = 8.6\times 10^{6}\,{\rm pc}$$

4. (a) **Parallax**: Well, the dimmer the better. Any star you can measure well enough that's close enough you can measure parallax from.

   Main-Seq. Fitting: Luminous and dim. You need to see enough of a range to be able to see the main sequence in a cluster.

   Cepheids: Only luminous; Cepheid variables are luminous stars.

   SNe: Only luminous; SNe are extremely luminous.

   (b) **Parallax**: Nearby stars, local region of Milky Way.

   Main-Seq. Fitting: Milky way galaxy scale.

   Cepheids: Milky Way Galaxy & nearby galaxies.

   SNe: Nearby galaxies & much of the Universe.

   (b) **Parallax**: Not at all; it depends only on our knowing the size of Earth's orbit about the Sun.

   Main-Seq. Fitting: Depends on having some stars with measured parallax so that, first of all, we can even construct the H-R diagram of luminosity vs. temperature and identify the main sequence at all, and second of all, so that we can figure out what the luminosity of a main-sequence star of each temperature is.

   Cepheids: Depends on parallax and/or main-sequence fitting, so that we can figure out the distance to at least a few Cepheid variables to know what luminosity corresponds to what period.

   SNe: Thermonuclear SNe are very rare. We are dependent on finding a few of them in galaxies with out distance measurements— ideally Cepheid distances— in order to calibrate their luminosity.

5. (a)

$$d = \frac{1}{p} = \frac{1}{0.38}\,{\rm pc}$$

$$\boxed{d = 2.63\,{\rm pc}}$$

   (b)

$$\frac{B_S}{B_\odot} = \frac{\frac{L_S}{4\pi\,{d_S}^2}}{\frac{L_\odot}{4\pi\,{d_\odot}^2}}$$

$$\frac{B_S}{B_\odot} = \left(\frac{L_S}{L_\odot}\right)\left(\frac{d_\odot}{d_S}\right)^2$$

$$L_S = \left(\frac{B_S}{B_\odot}\right)\left(\frac{d_S}{d_\odot}\right)^2 L_\odot$$

$$L_S = \left(\frac{1}{1.2 \times 10^{10}}\right)\left(\frac{2.63\,\text{pc}}{1\,\text{AU}}\right)^2 \left(\frac{206,265\,\text{AU}}{1\,\text{pc}}\right)^2 L_\odot$$

$$\boxed{L_S = 25\,L_\odot = 9.4 \times 10^{27}\,\text{W}}$$

(c) Look at the HR diagram in question 5. A star that is of spectral type A but only 25 times the luminosity of the Sun is well within the main sequence band.

(d) White dwarf. It's way the heck dimmer than Sirius A, but the same temperature; this puts it down with the white dwarfs on the HR diagram.