Astro 102 Review Problem Set #5
Solutions

1. If we find out that the supernova was hidden behind dust, then we realize that we have overestimated the *distance* to the supernova. That means we've overestimated the *lookback time*. We measured the redshift right, so we calculated a certain amount of expansion in *more time than was right*. That's a *slower* expansion rate, and therefore we've ***underestimated the expansion rate of the Universe***.

2. The Hubble Time t_H is the expansion timescale for the expansion. For the Universe to expand by 1% of its current size, it will take 1% of the Hubble Time, or $0.01\,t_H =$ 138 million years.

If you would rather do this with equations, you can say exactly the same thing as the previous paragraph with:

$$\frac{\Delta t}{t_H} = z = \frac{\Delta R}{R_0}$$

Where R_0 is the "size of the Universe" (e.g. the average distance between galaxies) now and ΔR is the change in the size. Thus, we have:

$$\begin{aligned}\frac{\Delta t}{t_H} &= \frac{0.01\,R_0}{R_0}\\ \Delta t &= 0.01\,t_H\end{aligned}$$

Giving us the same answer of 138 million years.

3. **(a)** Recall that the observed brightness is proportional to luminosity divided by distance squared:

$$B = \frac{L}{4\pi\, d^2}$$

Here, we have two objects: the Supernova, and Vega. We know that $B_{SN} = 1.6 \times 10^{-7}\, B_V$, so:

$$B_{SN} = \frac{L_{SN}}{4\pi\, {d_{SN}}^2} = 1.6 \times 10^{-7}\, B_V = 1.6 \times 10^{-7}\,\frac{L_V}{4\pi\, {d_V}^2}$$

We know L_{SN}, L_V, and d_V, so we can solve the equation:

$$\frac{L_{SN}}{4\pi\, {d_{SN}}^2} = 1.6 \times 10^{-7}\,\frac{L_V}{4\pi\, {d_V}^2}$$

Cross multiply:

$$L_{SN}\,(4\pi)\,{d_V}^2 = 1.6 \times 10^{-7}\, L_V\,(4\pi)\,{d_{SN}}^2$$

This gives ups:

$$d_{SN} = d_V \sqrt{\left(\frac{1}{1.6 \times 10^{-7}}\right)\left(\frac{L_{SN}}{L_V}\right)}$$

Now we can plug in:

$$d_{SN} = (7.76\,\mathrm{pc} \sqrt{\left(\frac{1}{1.6 \times 10^{-7}}\right)\left(\frac{5.8 \times 10^9\, L_\odot}{130\, L_\odot}\right)}$$

$$d_{SN} = 1.3 \times 10^8\,\mathrm{pc} = \boxed{130\,\mathrm{Mpc}}$$

(b) Use the expanding Universe equation:

$$z = \frac{d}{c\, t_H}$$

We want t_H, and we know everything else, so solve:

$$t_H = \frac{d}{c\, z}$$

You will save yourself some conversion if you use $c = 1$ lyr/yr instead of the usual m/s value. Then only one conversion is needed, 1 pc = 3.26 yr.

$$t_H = \left(\frac{1.3 \times 10^8\,\mathrm{pc}}{(1\,\mathrm{lyr\,yr^{-1}})\,(0.062)}\right)\left(\frac{3.26\,\mathrm{lyr}}{1\,\mathrm{pc}}\right)$$

$$t_H = 6.8 \times 10^9\,\mathrm{yr} = 6.8\,\text{billion years}$$

This is not the real value for our Universe, of course. . . .

(c) The distance to this galaxy in light-years is $(130\,\mathrm{Mpc})(3.26\,\mathrm{lyr}/1\,\mathrm{pc}) = 420$ Mlyr. That means that the lookback time is 420 million years. That means that the first explosion happened 420 million years ago; if the second explosion happened 200 million years after that, it happened $\boxed{\text{220 million years ago}}$. Of course, if we've just seen the first supernova, we won't *see* the second supernova for another 200 million years. (In reality, there will probably be lots of supernovae in this galaxy in between. Type Ia supernovae happen about once every 500 years in a big galaxy like ours.)

4. (a) Remember that cosmological redshift is the result of the fact that the wavelength of light expands at the same rate as the Universe; as such,

$$1 + z = \frac{\text{Size Now}}{\text{Size Then}}$$

Therefore the size of the universe then divided by the size of the Universe now is $1/1101 = 0.00091$. (A little less than one-thousandth the size.)

(b) If the linear size was 0.00091 the size now, the volume was $(0.00091)^3 = 7.5 \times 10^{-10}$ times the volume now.

Density is mass divided by volume. Since dark matter is just matter, it isn't created or destroyed, so in a given region of the Universe, the *mass* of the dark matter was the same as it is now, but the volume of that region was 7.5×10^{-10} what it is now. As such, the density was $1/7.5 \times 10^{-10} = 1.3 \times 10^9$ what it is now. Multiply that by the current density, and you get a density of $\boxed{3.2 \times 10^{-21}\ \mathrm{g/cm^3}}$. (Notice that while we refer to the Universe as "high density" at the time of the emission of the CMB, it's only in comparison to the Universe today; it's still *very* low density compared to the atmosphere of the Earth!

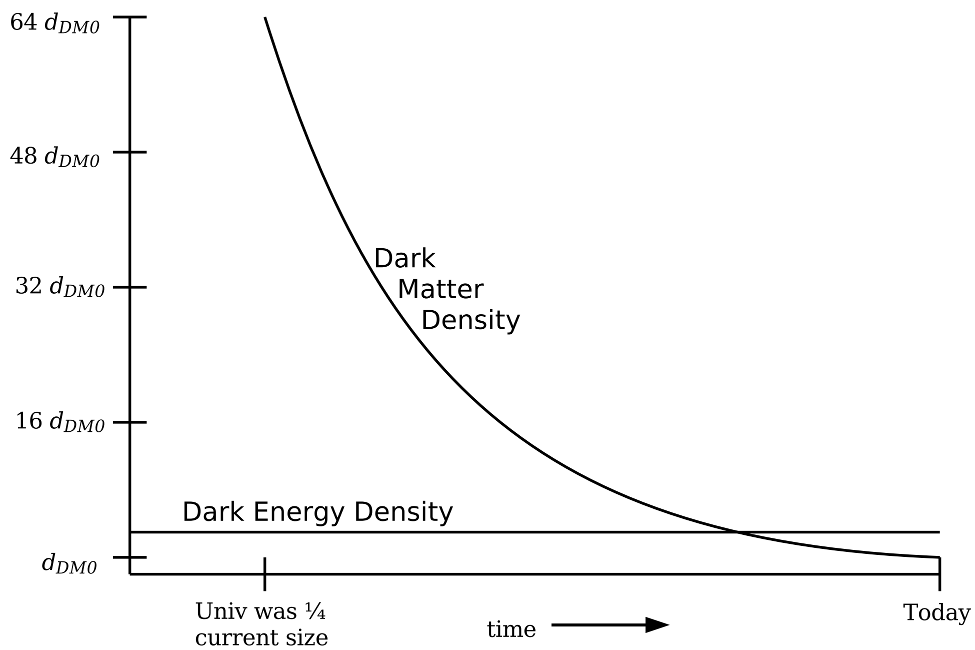
(c) Your first instinct is probably to repeat the calculation of the previous problem...but remember that vacuum energy has the property that *its density is constant*. If you have twice as much vacuum, you have twice as much vacuum energy.

As such, the density of Dark Energy back then was $\boxed{6.7 \times 10^{-30}\ \mathrm{g/cm^3}}$. Back then, the Dark Energy density was *much* less than the Dark Matter density!

One day I will figure out if I'm always supposed to capitalize Dark Matter.

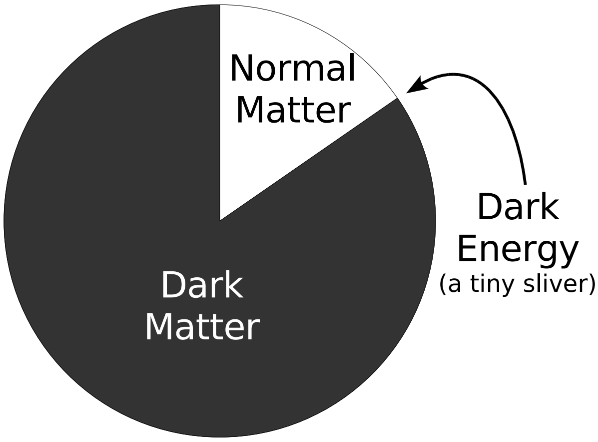
(d) Here, we can repeat the calculation from (b). The density of normal matter was be 1.3×10^9 what it is now, or $\boxed{6.5 \times 10^{-22}\ \mathrm{g/cm^3}}$. Notice that the ratio of normal matter to dark matter stays the same (about 1/5).

(e)



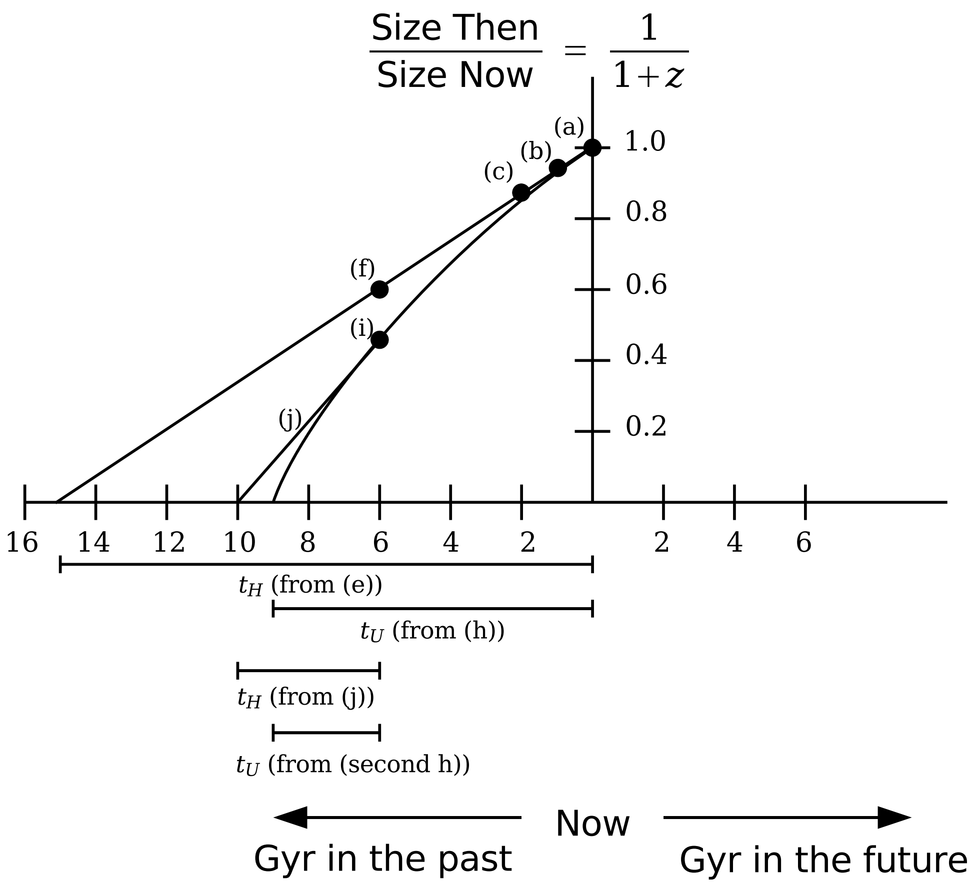
Here's how I figured this out. First, d_{DM0} is today's density of dark matter. Dark matter should be going up as the size of the Universe cubed (since that's how volume goes). Thus, when the Universe was 1/4 today's size, the density should have been $4^3 = 64$ times higher. Dark Energy, meanwhile, has a constant density; today's Dark Energy density is about three times today's Dark Matter density.

(f)



5. The right answer is (c). There are two things to notice. First, the galaxies are getting farther apart, but they are not expanding themselves. Second, there was more expansion between the second and third times than between the first and second times; that's an accelerating expansion.

6. Here's the plot. Discussion is below.



(b) Note that for $z = 0.067$, we have $1/(1+z) = 0.94$. The lookback time for this galaxy is 1 billion years.

(c) The lookback time is easy: it's 2 billion years. For the redshift, consider that the light took twice as long to reach us from this galaxy as it did from the galaxy in (b). As such, the Universe will have expanded twice as much, giving us $z = 0.134$. From that, we can figure out $1/(1+z) = 0.88$.

(d) Extrapolating a line that far is always hard— you can get quite a variation in where it intersects while still going pretty well through the points— but it looks like we should get a t_H of about 15 billion years.

If we calculate a t_H from the data above, we get:

$$z = \frac{d}{c\,t_H}$$

$$t_H = \frac{d}{c\,z} = \frac{1 \times 10^9\,\mathrm{lyr}}{(1\,\mathrm{lyr/yr})\,(0.067)}$$

Here, I've put in the data for the galaxy of (b), and have used the most convenient form of c. That gives us $t_H = 14.9$ billion years for this problem, or really 15 billion years given the number of sig figs we have.

(j) t_H is obviously lower 6 billion years in the past as compared to today. It is *also* lower than the t_H we would have calculated in (f) (which would have been 9 billion years— can you explain why?).

(second h) If we're on the decelerating Universe, then it's clear that we always have $t_U < t_H$. However, looking at the plot, you see that 6 billion years ago, t_U/t_H was *bigger* (that is, t_H was a closer approximation of t_U) than it is today. In a decelerating Universe, the ratio t_U/t_H gets ever slower (that is, t_H goes up faster than the age of the Universe as a result of the deceleration).

(In a Universe with constant expansion rate, t_H and t_U go up at the same rate; can you explain why?)

Our real Universe is accelerating now, and was decelerating in the past, so the relationship between t_H and t_U is more complicated.