Astronomy 102, Spring 2003

Solutions to Review Problems from 2003 February 5

1. Chapter 4, Question 12 in the text: *Imagine you are riding in a car, tuned in to 790 on the AM radio dial. This station is broadcasting at a requency of 790 kilohertz (7.9 × 10⁵ Hz). What is the wavelength of the radio signal? You switch to the FM station at 98.3 on your dial. This station is broadcasting at a frequency of 98.3 megahertz (9.83 × 10⁷ Hz). What is the wavelength of this radio signal?*

The basic equation here is $\lambda f = c$, where λ is the wavelength, f is the frequency, and c is the speed of light (3.00×10^8 m/s). Thus, to find wavelength, we solve this to $\lambda = c/f$. Recall that 1 Hz = 1 s^{-1} = 1/1 s.

$$\frac{3.00 \times 10^8 \text{ m s}^{-1}}{7.9 \times 10^5 \text{ s}^{-1}} = 380 \text{ m}$$

$$\frac{3.00 \times 10^8 \text{ m s}^{-1}}{9.83 \times 10^7 \text{s}^{-1}} = 3.05 \text{ m}$$

2. Chatper 4, Question 15 in the text: *Refer to Question 13 in Chapter 3 about the hypothetical planet named Vulcan. If such a planet existed in an orbit 1/4 the size of Mercury's, what would be the averate temperature on Vulcan's surface? Assume that the averate temperature on Mercury's surface is 450K.*

(Not to be confused with Mr. Spock's Vulcan....) Refer to the equation in class we derived for thermal equilibrium of a planet around a class:

$$\frac{L_\odot}{4\pi d^2}\pi R_p^2(1-a) = 4\pi R_p^2 \sigma T^4$$

where $L_\odot$ is the luminosity of the sun, d is the distance of the planet from the Sun, R_p is the radius of the planet, a is the albedo of the planet, and T is the average surface temperature of the planet. That looks like a lot of letters, but you can understand it if you break it down and ask what each piece means. The left side tells you the rate at which the planet is *heating up* by absorbing the Sun's rays. $L_\odot/(4\pi d^2)$ is the *flux* of the Sun's light at the planet, a distance d from the Sun. πR_p^2 is the cross-sectional area of the planet, like the aperture of yoru telescope, that tells you how much of that flux the planet is intercepting. Finally $(1-a)$ is the fraction of the energy that gets absorbed (since a is the fraction that gets reflected). On the right side, you have the rate that the planet is *cooling off* because it's radiating. σT^4 tells you the rate at which one square meter of an object at temperature T radiates energy, and $4\pi R_p^2$ is the total surface area of the planet (i.e. the total number of square meters that are radiating). At thermal equilibrium, the left and right sides are the same: the rate the planet is heating up must be the same as the rate at which it is cooling off. (If they weren't the same, then the temperature would either go up or go down.)

Take this equation, and divide both sides by $4\pi R_p^2$, and you get:

$$\frac{L_\odot}{16\pi\sigma d^2}(1-a) = T^4$$

If you know T^4, you can take a square root twice to get T. Now, we could look up $L_\odot$. We don't know d, but we could figure it out by looking up the orbit of Mercury and dividing by 4. Finally, we can look up σ, which is just the Steffan-Boltzman constant and always has the same value. That would give us everything we need to know to plug in and calculate T.

However, consider another approach. We know what T is for Mercury, and we know that the two planets (Mercury and the fictitious Vulcan) are both orbiting the same Sun. So write down the equation, using d_V as the distance from the Sun to Vulcan, and d_M as the distance from the Sun to Mercury, and divide the two equations by each other:

$$\frac{\frac{L_\odot}{16\pi\sigma d_V^2}(1-a_V) = T_V^4}{\frac{L_\odot}{16\pi\sigma d_M^2}(1-a_M) = T_M^4}$$

$$\frac{L_\odot 16\pi\sigma d_M^2(1-a_V)}{L_\odot 16\pi\sigma d_V^2(1-a_M)} = \frac{T_V^4}{T_M^4}$$

Cancel out all the stuff that appears on both the top and the bottom; for simplicity, let's assume that $a_V = a_M$, so we can cancel the $(1-a)$ terms:

$$\frac{d_M^2}{d_V^2} = \frac{T_V^4}{T_M^4}$$

That starts to look very simple. Rewrite this as:

$$\left(\frac{d_M}{d_V}\right)^2 = \left(\frac{T_V}{T_M}\right)^4$$

Take the square root of both sides, twice, and you get:

$$\sqrt{\frac{d_M}{d_V}} = \frac{T_V}{T_M}$$

Since $d_M = 4d_V$, and $\sqrt{4} = 2$, we know that $T_V/T_M = 2$, so therefore $\boxed{T_V = 900K}$. Note that if you came out with an answer that had a temperature *less* than Mercury's you should be suspicious: Vulcan is closer to the Sun than Mercury! It always pays to look at the end and make sure your answer makes sense with what is obvious.

3. Chatper 4, Question 16 in the text: **Oops!** I also assigned this as homework on homework set #2. Wait for that to see the soultion.

4. *Stars similar to the Sun, late in their life, turn redder in color; however, they also become much more luminous (emit more total energy per second). From just this information and what you know of thermal (blackbody) radiation, what can you conclude about the nature of these stars late in their life?*

You know from blackbody radiation that objects which are redder emit less energy per square meter—that is, if you have two objects which are of the *same size*, the redder one emits less energy per second. Therefore, if stars get redder late in life, but also get more luminious, they must also be getting larger, that is, their radius (and therefore volume and, most importantly, surface area) must increase.

5. (a) *The Sun is 93 million miles away. How far back in time are we looking when we observe the sun?*

(b) *The Andromeda Galaxy (a nearby large galaxy) is 0.8 Mpc (that's 0.8 megaparsecs, or 800,000 parsecs) away. How far back in time are we looking when we observe the Andromeda Galaxy?*

(a)

$$93 \times 10^6 \text{mi} \left(\frac{1.609 \times 10^3 \text{ m}}{1 \text{ mi}}\right) = 1.50 \times 10^{11} \text{ m}$$

We know the distance, and we know the speed (the speed of light). Divide distance by speed to get time; if you forget which way to divide it, just do the way that gets the units right:

$$\frac{1.50 \times 10^{11} \text{ m}}{3.00 \times 10^8 \text{ m s}^{-1}} = 500 \text{ s} = 8.3 \text{ m}$$

(Bringing up the memory of exam 1....)

(b) This is the same conversion, only you need to know that $1 \text{ pc} = 3.26 \text{ light}-\text{years} = 3.09 \times 10^{16}$ m. Thus, the distance to the Andromeda Galaxy is:

$$0.8 \times 10^6 \text{ pc} \left(\frac{3.09 \times 10^{16} \text{ m}}{1 \text{ pc}}\right) = 2.47 \times 10^{22} \text{ m}$$

(I'm keeping more sig figs than I will quote in the final answer, just so I don't lose precision in the calculations.) As before:

$$\frac{2.47 \times 10^{22}\ \text{m}}{3.00 \times 10^{8}\ \text{m s}^{-1}} = 8.2 \times 10^{13}\ \text{s} = 2.6\ \text{million years}$$

The Andromeda Galaxy is the most distant object you can readily see with your naked eye (i.e. without binoculars or a telescope). You can't see it from a light-polluted area like downtown Nashville, but it's not too hard to see if you get under a good, dark sky. It's in the constellation Andromeda (unsurprisingly), though is easier to find starting with the Great Square of Pegasus. You can find it (labelled "M31") on your SC001 star chart. It's best to look for this object in the late fall or early winter, as that's when it's high in the evening sky. Right now (mid-February), it sets not too long after the sun. If you do find this galaxy, and regard it with your naked eye, realize that you're not seeing the galaxy as it is now, but rather as it was neary 3 million years ago. (What were *you* doing 3 million years ago?)

6. ***[More challenging]*** *Astronomers use the term* absolute magnitude *to describe the magnitude of a star as it would be observed from a distance of 10pc, and* distance modulus *to describe the difference in magnitude between the star as observed and its absolute magnitude. Remembering the definition of magnitude as*

$$m = -2.5 \log\left(\frac{f}{f_0}\right)$$

where f is the flux of a star, and f_0 is a constant, what would be the distance modulus of a star which is 1kpc away?

This one requires remembering how to play with logs. However, you can get started even if you don't remember how to do that. Where to begin? Well, we're looking at the flux of objects observed at different distances. Where have we talked about flux and distance relating to each other? Recall (or look up in your notes) that the observed flux of an object is

$$f = \frac{L}{4\pi d^2}$$

where L is the luminosity of the object (total energy radiated per second) and d is the distance to the object. If you have two stars of the same luminosity ($L_1 = L_2$; since they're the same let's just call it L), but one is at distance d_1 and the other is at distance d_2, the ratio of their fluxes is:

$$\frac{f_1}{f_2} = \frac{\frac{L}{4\pi d_1^2}}{\frac{L}{4\pi d_2^2}}$$

(Why take a ratio of the fluxes? Well, we're trying to compare the brightnesses of two stars, and generally with fluxes that means taking the ratio of how bright one is to the other. It also helps to remember that *a difference of magnitudes (log fluxes) corresponds to a ratio of fluxes.* Since we're eventually going after a difference of magnituces, we should expect that a ratio of fluxes may come into it at some point.) Cancelling out the things that appear on both the top and the bottom of the right side:

$$\frac{f_1}{f_2} = \frac{d_2^2}{d_1^2} = \left(\frac{d_2}{d_1}\right)^2$$

For the problem in question, we want to know the difference in magnitude between a star at 10pc and 1kpc. Thus, it makes sense to set $d_2 = 10$pc and $d_1 = 1000$pc (because 1kpc=1000pc). This tells us:

$$\frac{f_1}{f_2} = \left(\frac{10}{1000}\right)^2 = 10^{-4}$$

How are we going to get a magnitude out of this? Well, notice in the magnitude equation above that f is divided by f_0. To make our equation look more like this, let's divide both the top and bottom of the left side by f_0:

$$\frac{f_1/f_0}{f_2/f_0} = 10^{-4}$$

Take a log of both sides:

$$\log\left(\frac{f_1/f_0}{f_2/f_0}\right) = -4$$

where I've used the fact that $\log(10^x) = x$ on the right side. On the left side, you need to know (or look up) that $\log(x/y) = \log(x) - \log(y)$, so we have:

$$\log\left(\frac{f_1}{f_0}\right) - \log\left(\frac{f_2}{f_0}\right) = -4$$

Now multiply both sides by -2.5, and we have;

$$-2.5\log\left(\frac{f_1}{f_0}\right) \; - \; -2.5\log\left(\frac{f_2}{f_0}\right) = 10$$

Aha! Now this looks just like the magnitude equation, so we have:

$$m_1 - m_2 = 10$$

That's exactly what we are looking for; since we set $d_2 = 10$ pc, m_2 is the magnitude of the object at 10pc; similarly, m_1 is the magnitude of the object at 1kpc. The *distance modulus* is the difference in the magnitude measured by an observer at the real distance and a hypothetical observer at 10pc, so what we have calculated here is exactly the answer: $\boxed{\text{the distance modulus for 1kpc is 10}}$. Note that that is just "10"– there are no units on magnitudes. Notice also that the difference is positive, i.e. $m_1 > m_2$, or the magnitude of a star observed at 1kpc ($m1$) is greater than the magnitude of a star observed at 10pc ($m2$). Remember that *greater magnitudes indicate fainter objects.* You expect the further object to be fainter, so the fact we got a positive difference makes sense.

This problem is more mathematically involved than anything I would ask on a test; however, it is useful to be able to follow what was done. Except for needing to take the log identity as given, you should be able to understand and perform each step.