Astronomy 102, Spring 2003

Solutions to Review Problems from 2003 February 12

1. Chapter 12, Question 1 in the text: *To know certain properties of a star, you must first determine the star's distance. For other properties, knowledge of distance is not necessary. Into which category would you place each of the following properties: size, mass, temperature, color, spectral type, and chemical composition? In each case, state your reason(s).*

Properties requiring knowledge of the distance: size and mass. For size, if (for a very nearby star) you measure the angular diameter, you will need the distance in order to perform the trigonometry (probably using the small angle formula) to figure out the physical diameter. If instead you're using the formula $L = 4\pi R^2 \sigma T^4$, if you know the temperature T and the luminosity L, you can calculate the size R. You can get L by solving $F = \frac{L}{4\pi d^2}$, where F is the flux you measure— but you then also need d, the distance, in order to calculate L. For mass, if it's a binary star, you can measure the period of the orbit without knowing the distance, but again to get the physical semi-major axis of the orbit from the angular separation of the two stars, you need to know the distance to the system.

Properties *not* requiring knowledge of the distance: temperature, color, spectral type, and chemical compostion. All of these can be measured from the spectrum of the star. For each of them, you will be comparing the flux at one wavelength to the flux at another wavelength. Although the value of the flux at any given wavelength depends on the distance you are from a given star, the ratio between fluxes at different wavelengths does not. Color is just the ratio of fluxes at two different wavelengths, and from that you can figure out temperature. The chemical composition comes from looking at the strenghts of absorption lines— i.e. the ratio of the flux on the absorption line to the flux just off of the absorption line. Finally, spectra type comes from looking at what absorption lines are there and how strong they are, as well as from the color of the star.

2. Chapter 12, Question 4 in the text: *: Although we tend to think of our Sun as an "average" main sequnece star, it is actually hotter and more luminous than average. Explain.*

Stars cooler and less luminous than the sun are far more common than the more luminous, hotter stars. When you average together temperatures and luminosities of all stars, there will be more weight for the cooler, dimmer stars, and the average star will therefore be cooler and less luminous than the Sun.

3. Chapter 12, Question 10 in the text: *: Logarithmic (log) plots show major steps along an axis scaled to represent equal factors, most often factors of 10. Why, in astronomy, do we sometimes use a log plot instead of the more conventional linear plot?*

The stretching of the range that a log plot gives you allows you to see what's going on with the data. Consider, for example, the HR diagram (e.g Figure 12.15a in the book). If we want to plot all of the stars in this figure, we'd need to make sure the range goes all the way up to at least 10^3 $L_\odot$. Notice, however, that most of the points on this plot have a luminosity $< 10^1$ $L_\odot$. This means that most of the points occupy a range of luminosity which is less than 1/100th of the total range. On a linear plot running from luminosity 0 to 10^3 $L_\odot$, most of the points would be squished down onto the bottom hundredth of the plot, with the few reamining points stretched out over the rest of the vertical space of the plot. It would be much harder to see what is going on and what range of luminosities are present.

In general, when some physical quantity varies by factors— e.g., many factors of 10— you will often find that it's easier to see all the information you want to see on a log plot rather than a traditional linear plot. Only when the physical quantity varies by approximately even steps (as opposed to factors) does it make sense to use a conventional linear plot— for instance, when plotting the RA and decs of stars.

4. Chapter 12, Questions 13 and 14 in the text: *: Sirius, the brightest star in the sky, has a parallax of 0.379 arcseconds. What is its distance in parsecs? In light-years?*

Sirius is 22 times more luminous than the Sun, and Polaris (the "Pole Star") is 2,350 times more luminous than the Sun. Sirius appears 23 times brighter than Polaris. what is the distance of Polaris in light-years?

First, the distance to Sirius. We know $d = 1/p$ where p is the parallax in arcseconds and d is the distance in light-years.

$$d = \frac{1}{p} = \frac{1}{0.379} = 2.64 \text{ parsecs}$$

Knowing that 1 pc = 3.26 light-years, this is easy enough to convert:

$$2.64 \text{ parsecs} \left(\frac{3.26 \text{ light} - \text{years}}{1 \text{ parsec}} \right) = 8.60 \text{ light} - \text{years}$$

This next section is a bit trickier, but is similar to things on the previous homework set in that you are comparing the distance to two different objects. First of all, how do you get distance when you know luminosity and how bright something appears? Remembering that "how bright it appears" is the same as flux, the place to start is:

$$F = \frac{L}{4\pi d^2}$$

where F is flux, L is luminosity, and d is distance. This equation relates the three things we want: two we know (F and L), and the one we're trying to figure out d. However, we can't just plug things in directly. We *could* look up the luminosity of the Sun, and figure out L for Polaris. However, we aren't given a number for F anywhere, only the comparison between Sirius and Polaris.

So, what we know is (using the subscript S for quantities describing Sirius, and the subscript P for quantities describing Polaris):

$$L_S = 22L_\odot$$

$$L_P = 2350L_\odot$$

$$F_S = 23F_P$$

$$d_S = 8.60 \text{ light} - \text{years}$$

The flux/luminosity/distance equation must apply for each star:

$$F_S = \frac{L_S}{4\pi d_S^2}$$

$$F_P = \frac{L_P}{4\pi d_P^2}$$

At this point, there are a few ways we could proceed. We could divide the two equations by each other; that would give us ratios of things we know in terms of each other. Alternatively, we could start with $F_S = 23F_P$, and substitute in for F_S and F_P:

$$\frac{L_S}{4\pi d_S^2} = 23\frac{L_P}{4\pi d_P^2}$$

Substitute in for L_S and L_P:

$$\frac{22L_\odot}{4\pi d_S^2} = 23\frac{2350L_\odot}{4\pi d_P^2}$$

Now we've got an $L_\odot$ on both sides, so we can divide by it. Similarly, multiply both sides by 4π:

$$\frac{22}{d_S^2} = \frac{54050}{d_P^2}$$

(where the 54050 comes from multiplying 23 by 2350). Aha! Now we've got an equation with only one unknown; solve for that unknown:

$$d_P = \sqrt{54050\frac{d_S^2}{22}} = 50d_S$$

$$d_P = 430 \text{ light} - \text{years}$$

Think about this: Polaris appears *dimmer* than Sirius, but is intrinsically *more luminous* than Sirius. Thus, Polaris had better be further away. Sure enough, we've calculated a greater distance for Polaris than for Sirius. (If we had calculated a smaller distance, we'd know we'd done something wrong.)

5. *Consider two telecopes: the lab telescope, whose aperture has a diameter of 8 inches, and the Hale telescope at Palomar Observatory, whose aperture has a diameter of 200 inches.*

    (a) If you measure the flux of a star with the Hale telescope to be f, what would you measure the flux of the same star to be with a lab telescope? (In other words, what is the ratio of the flux observed by the Hale telescope to the flux observed from the same star by a lab telescope.)

    (b) How long will it take the lab telescope to collect the same total amount of energy as the Hale telescope collects in one second?

(a) You will measure the *same* flux with the two telelscopes. Remember that flux is the amount of energy arriving *per square meter* at the earth; this flux has nothing to do with what instrument you use to measure it. Thus, even though one telescope has more total surface area (and thus will collect more total area) than the other, each is pulling from the same flux. Think about collecting rainwater: if you have two glasses, one twice as wide as the other, the wider glass will collect more total water in a given period of time. However, the rainfall rate is the same regardless of which glass you happen to be looking at.

(b) Flux is energy per second per area, thus the total energy collected in one second by a telescope is the flux times the collecting area of the telescope (πr^2). The energy per time collected by the Hale telescope (diameter $200''$, so radius $r_H = 100''$) is:

$$\frac{E_H}{t_H} = F \, \pi \, r_H^2$$

Similarly, the energy per time collected by the lab telescope (diameter $8''$, so radius $r_L = 4''$) is:

$$\frac{E_L}{t_L} = F \, \pi \, r_L^2$$

We're talking about a case where the total energy collected is the same in each case ($E_H = E_L$), so solve each of these equations for total energy:

$$E_H = F \, \pi \, r_H^2 \, t_H$$

$$E_L = F \, \pi \, r_L^2 \, t_L$$

Set $E_H = E_L$:
$$F \, \pi \, r_H^2 \, t_H = F \, \pi \, r_L^2 \, t_L$$

We want to know how much longer it will take the lab telescope to collect the same energy as the Hale telescope collects in 1 second, so solve this equation for t_L:

$$t_L = \frac{r_H^2}{r_L^2} t_H$$

Plug in the numbers ($r_H = 100''$ and $r_L = 4''$), and you get

$$\frac{t_L}{t_H} = 625s$$

In other words, it takes more than *ten minutes* with the lab telescope to collect the amount of energy that the Hale telescope collects in one second. This is why astronomers like to use big telescopes when they can.

6. *Consider two stars of the same luminosity. Star A has a temperature which is twice the temperature of Star B. What is the ratio of the radius of Star B to the radius of Star A?*

You need to relate radius, luminosity, and temperature. That equation, from class and from the book, is:

$$L = (\sigma T^4)(4\pi R^2)$$

Thus, we have two equations for the two stars:

$$L_A = (\sigma T_A^4)(4\pi R_A^2)$$

$$L_B = (\sigma T_B^4)(4\pi R_B^2)$$

We are told the two luminosities are the same, so we can set $L_A = L_B$:

$$(\sigma T_A^4)(4\pi R_A^2) = (\sigma T_B^4)(4\pi R_B^2)$$

Divide the constants out of both sides:

$$T_A^4 R_A^2 = T_B^4 R_B^2$$

Solve this for the ratio we want:

$$\frac{R_B}{R_A} = \sqrt{\frac{T_A^4}{T_B^4}} = \left(\frac{T_A}{T_B}\right)^2$$

We know $T_A/T_B = 2$, so therefore $\boxed{R_B/R_A = 4}$.