Astronomy 102 Spring 2004: Exam 1 Solutions and Commentary

1. (c) Celestial Equator

2. (d) The tilt of the axes, as that is the reason we have seasons in the first place. Substantially changing the eccentricity of the orbit of the Earth *would* affect the seasons, but since the Earth's orbit is currently close to circular, changing the eccentricity would mean making the distance betwen the Earth and the Sun more variable— which would if anything make the effect of the seasons greater.

3. (e) 8.3 minutes. This was just a unit conversion and significant figures problem. The speed of light is given on the front of the test.

$$(9.3 \times 10^7 \text{ miles}) \left(\frac{1609 \text{ m}}{1 \text{ mile}}\right) = 1.4964 \times 10^{11} \text{ m}$$

$$\frac{1.4964 \times 10^{11} \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 4.9879 \times 10^2 \text{ s}$$

$$4.9879 \times 10^2 \text{ s} \left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 8.3132 \text{ min}$$

But! Even though keeping all the digits returns 8.3132 minutes as the answer, (d) is *wrong* because we only have the distance between the Earth and the sun to two significant figures. Five is clearly too many, thus (e) best represents what we really know about the time it takes light to reach the Earth from the Sun.

4. (d) It is falsifiable. The other three are good for scientific theories that are likely to be right, but not necessary. Although scientists are suspicious of theories that contradict known wisdom, history is full of succesful theories that did; the Copernican model of the solar system is one example. Occam's Razor and the Cosmological Prinicple are guiding ideas which help you select which thoery is more likely to be right, but not necessary for a useful theory. Science works by experiement and by empirical evidence. If a theory makes no predictions which can in principle be tested by experiment or observation, then it is not a useful theory for science. It may be that we are many years or deacdes off from testing the theory, but it must have some consequence for what we would observe.

5. (c) Exactly 12 hours. Things north of the equator are up longer, things south of the equator are up less time, for an observer in the Northern hemisphere.

6. (a) We talked about this in class; the same face of the moon always faces the *Earth*; thus, the rotational period is synchronous with the period of the Moon's orbit around Earth. Over the course of a month, the moon will go through one complete rotation and expose all sides of itself to the Sun.

7. (b) Up and to the right. See the "Celestial Sphere" animation I showed in class, or the drawings on the board in class on Wed. Jan 29 to explain the "Moon Horns" problem.

8. (e) Smaller orbits means faster motion. There are a couple of ways to figure this out. The first is to remember the plot I drew in class of "Keplerian motion", where the velocity went *down* with increasing distance. The second is to remember the two laws $F = GMm/R^2$ and $F = mv^2/R$, put them together, and see that $v \propto 1/\sqrt{R}$. The third is to remember Kepler's Third Law (which was also on the front of the test), $P^2 \propto A^3$. As A goes up, P goes up *faster* ($P \propto A^{3/2}$). This means that as the radius gets larger (e.g. by looking from one planetary orbit to the next further out), the time takes to go around a circle gets larger by a greater factor. If the time it takes to travel is increasing faster than the distance being travelled, then the speed must be going down.

9. (a) At the equator. You can see both the North and South poles; over the course of the year, at (say) midnight, all Right Ascensions will pass through your meridian, and thus you will have been able to see the entire sky.

(b) One half of the sky. Everything at decliation $\delta < 0$ is below your horizon, and stays that way.

10. This problem required very similar reasoning to problems on the first homework set which were discussed in class.

(a) The moon rotates at the same rate it orbits— or, one rotation per month.. One rotation=one day, and we get one sunrise per day on Earth. Thus, you get one sunrise per month. (This is closer to 29 days than 27 days, although I gave full credit for 27 days since we haven't talked about the subtlety of the difference in class. See the text for the reason why there's only one sunrise every 29 days even though the orbital period of the moon is 27 days.)

(b) Never; because the same face of the moon always faces the Earth, the Earth doesn't appear to move in the moon's sky. Cf: homework set 1.

11. Answer: no change. A lot of people got this; a lot of people also got this one wrong. This is one of the most common popular misconceptions about Astronomy, that black holes suck up everything in the vicinity. They do no such thing; the point of this problem was that with what you know in the class already, you can figure out that this can't be the case even though you may know nothing specifically about black holes.

As I mentioned in class, all you needed to know about Newton's law of gravitation was that more mass means more force, and more distance means less force. If the Sun were to collapse into a black hole, but still have the same mass as before it collapsed, the gravitational force *does not change*. The distance is the same; the Sun hasn't moved. The mass is the same, as stated in the problem. Thus, at the distance the Earth is from the Sun, the effect of gravity from a normal star and of a black hole of the same mass is *indistinguishable*.

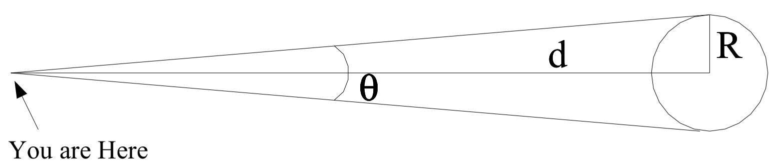
A few people said that the orbit would get smaller becuase there was now a mass of $2M$ at the center of the solar system. I realized this was from a misreading of a problem: they assumed that the black hole of one solar mass appeared and gobbled up the Sun, and then figured that the mass of both were still there. This is not what I intended, but I can see where the misreading came from. If the gravitational consequences were reasoned clearly and correctly, I gave substantial partial credit.

A few people also figured that the sun was just *gone*. If they misread the problem this way, *and* clearly and correctly described that without the Sun's gravity to hold the Earth in orbit, it would fly off in a straight line, I gave half credit. (The problem statement does clearly indicate that the black hole hass mass!)

I gave *no* credit to those who argued that the Earth would be sucked in due to the greater gravity of the black hole somehow resulting from its greater density. That is wrong, and was also the point of the problem...

12. At latitude $\delta = 23.5°$, the celestial equator is at altitude $90° - 23.5° = 66.5°$. December 22 is the middle of Winter, or the Winter Solstice. The declination of the Sun on this day is $\delta = -23.5°$, or $23.5°$ below the equator. Thus, its altitude is $66.5° - 23.5° = 43°$.

13. This problem is basically identical to one I did in class on the second day— only there, given the apparent angular size of Jupiter, I calculated its actual radius. One way to see the answers to some "word problems" like this is to try to draw the picture of what the problem is really talking about. We are on Earth; we are looking at Betelgeuse 650 light-years away. What are we looking at? The diameter, i.e. the distance between two points on opposite sides of the star. Draw the picture:



In the picture, R is the radius of Betelgeuse, d is the distance to Betelegeuse, and θ is the apparent angular *diameter* of Betelgeuse. (Thus, $\theta/2$ is the apparent angular *raduis*.) This is a right triangle, so:

$$\tan\frac{\theta}{2} = \frac{R}{d}$$

What's more, because d is so much larger than R, we can use the small angle formula. (In any event, you've all probably seen Betelgeuse in the lab by now, and you know that it just doesn't look that big.) Before we can divide R and d, though, we first have to get them in the same units.

$$R = 500 \text{ Solar Radii} \left(\frac{6.96 \times 10^5 \text{ km}}{1 \text{ Solar Radius}}\right) = 3.48 \times 10^8 \text{ km}$$

$$d = 650 \text{ light} - \text{years} \left(\frac{9.461 \times 10^{12} \text{ km}}{1 \text{ light} - \text{year}}\right) = 6.15 \times 10^{15} \text{ km}$$

Now we're ready to divide them:

$$\frac{\theta}{2} \approx \tan\frac{\theta}{2} = \frac{3.48 \times 10^8 \text{ km}}{6.15 \times 10^15 \text{ km}} = 5.66 \times 10^{-8} \text{ radians}$$

Remember that the small angle formula works for the angle in radians! Thus, we need to convert it to arcseconds:

$$5.66 \times 10^{-8} \text{ radians} \left(\frac{206265''}{1 \text{ radian}}\right) = 0.0117''$$

Note that here, 0.0117″ means 0.0117 *arcseconds*, **not** 0.0117 *inches*! This is also $\theta/2$, so multiply it by 2 to get the final answer, to two sig figs (which is already one too many) of:

$$0.023''$$

14. (a) Look at the picture on the front of the exam. There, you see that the furthest point is $(1+e) \times A$. We know e, that's the eccentricity, but we don't know A, the semi-major axis, for that wasn't given; thus we're stuck. But, we do know the closest distance, which from the same picture is $(1-e) \times A$, so we can use that to figure out the semi-major axis:

$$(1-e) \times A = 0.5 \text{ AU}$$

$$(1-0.99) \times A = 0.5 \text{ AU}$$

$$A = \frac{0.5}{0.01} \text{ AU} = 50 \text{ AU}$$

Now that we know both A and e, we're in a position to calculate the furthest distance:

$$\text{furthest} = (1+e) \times A = (1+0.99) \times (50 \text{ AU}) = 1.99 \times 50 \text{ AU} = 99.5 \text{ AU}.$$

That was an acceptable answer, although really it has too many significant figures. We only had the smallest distance to one significant figure in the first place, and we've multiplied by that number. Thus, 100 AU is really a better answer.

(b) From Kepler's third law, we have

$$P^2 = A^3$$

where A is in AU and P is in years. We know $A = 50$ AU (*not* 100 AU!) from the previous part, thus we can calculate:

$$P(\text{years}) = \sqrt{A^3} = \sqrt{(50 \text{ AU})^3} = 353 \text{ years}$$

to three significant figures— which is too many, of course, so **350 years** or even **400 years** would be a better answer. I gave full credit for answers that used up to three significant figures, but more than that and it was clealry getting ridiculous given that you started with a one-significant-figure distance of closest approach.