Astronomy 102, Spring 2003

Homework Set 2 Solutions

1. *The moon and all of the planets appear to go through phases, meaning that most of the time we see only a part of their surface. The Sun never goes through phases. Explain why the Sun is different from the Moon and the planets.*

When you see the phases of the moon or another planet, you're looking at reflected light from the sun. The side of the moon or planet facing the sun is "lit up"; depending on the angle you look at the planet, you'll see all, some, or none of the lit side, giving rise to different phases. The sun, in contrast, emits its own light, and as such won't have a dark side.

Note that the other planets actually *do* emit their own light! We know that so long as a thermal body isn't at an absolute zero temperature, it is emitting. However, that emission is so much less than the Sun's emission (and, it's mostly in the infrared) that we don't really see it.

Some of you mentioned the fact that the planets all orbit the Sun. This is true, and it's one way the Sun is different; it is not, however, relevant to making the phases. Even if the Sun were very light and orbited the Earth, if it were as bright as it is it could still cause phases in other bodies such as the moon. The fact that the Sun emits light is all that's relevant. If you stated that the relative gravity was part of the cause, I took a few points off.

A couple people still seem to have the misconception that the moon shows phases because of shadows *cast on* the moon. This is incorrect! It's just a matter of looking at the moon and seeing part of it lit by the Sun, part of it not lit. If you stated that cast shadows were the cause of phases, I took off a few points; you should review the earlier matieral from the course! If you used the word "shadow" but were clear that you were just talking about the side that doesn't get lit, that was fine; if you were vague about the word "shadow", I took off one point. Because this is a *very* common misconception about the source of the moon's phases, it's important to understand.

2. *Chapter 3, Question 7: Imagine a planet moving in a perfectly circular orbit around the Sun and, because the orbit is circular, the planet is moving at a constant speed. Is the planet experiencing acceleration? Explain.*

There are two ways to answer this. First: acceleration means change in speed *or direction.* Speed isn't changing, but to move in a circle the direction of the planet's motion must change. Thus, the planet is acceleration.

Second: there is a net force (due to the gravitational attraction of the Sun) on the planet. Whenever you have a net force, you have an acceleration, therefore there is an acceleration towards the Sun.

3. *Chapter 3, Question 17: Weight refers to the force of gravity acting on a mass. We often calculate the weight of an object by multiplying its mass by the local acceleration due to gravity. The value of gravitational acceleration on the surface of Mars is 0.39 times that on Earth. Assume your mass is 85 kg. Then your weight on Earth is 8.33 N (8.33 N = mg = 85 kg × 9.8 m/s^2). What would your mass and weight be on Mars?*

Mass doesn't depend on location, so your mass is still $\boxed{\text{85kg}}$ on mars.

Weight is mg:

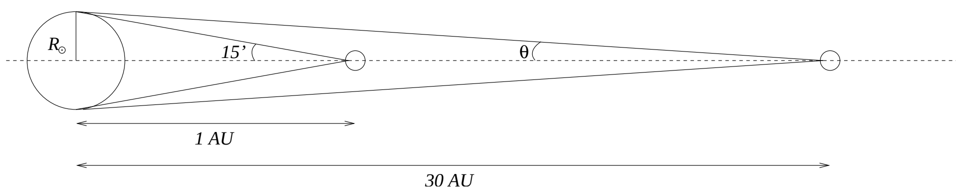
$$\text{mg} = (85\ \text{kg})(\ (0.39)(9.8\ \text{m s}^{-2})\ ) = \boxed{325\ \text{N}}$$

There are really only two significant figures in this problem; if you reported an answer to more than three, I docked you.

4. *Chapter 4, Question 11 a and c in the text: In our sky, the angular diameter of the Sun is about 30 minutes of arc. Now imagine that you have been transported to Neptune, 30 AU from the Sun.*

(a) *What would be the angular diameter of the Sun as seen in Neptune's sky?*

(c) *How bright woudl the Sun appear compared to its brightness as seen from the Earth?*

(a) 

$$\left(\frac{R_\odot}{1\ \text{AU}}\right) = \tan 15'$$

Note that the 15' is there because it's *half* of the angular diameter of the Sun, to get it to work in the right triangle as drawn. It's tempting to use the small angle formula, but we have to remember that only works for radians:

$$15'\left(\frac{60''}{1'}\right)\left(\frac{1\ \text{rad}}{206265''}\right) = 0.00465\ \text{rad}$$

It's also important to keep at least ane xtra digit or two beyond the significant figures of the problem for intermediate calculations!

Having this, we can write:

$$\left(\frac{R_\odot}{1\ \text{AU}}\right) \approx 0.00465$$

We could solve this for $R_\odot$, but instead let's take a shortcut. We also know, from the other triangle in this problem:

$$\left(\frac{R_\odot}{30\ \text{AU}}\right) = \tan\theta \approx \theta$$

Divide the second equation by the first equation:

$$\frac{1/30\ \text{AU}}{1/1\ \text{AU}} = \frac{\theta}{0.00465}$$

$$\frac{1\ \text{AU}}{30\ \text{AU}} = \frac{\theta}{0.00465}$$

$$\theta = \frac{0.00465}{30} = 0.000155\ \text{radians}$$

It would be more interesting to have this in arcminutes, so convert it back:

$$0.000155\ \text{radians}\left(\frac{206265''}{1\ \text{radian}}\right)\left(\frac{1'}{60''}\right) = 0.53'$$

This is only *half* of the angular diameter, so the full angular diameter is $\boxed{1'}$.

(b) Although we haven't discussed this, the human eye can resolve about $0.5'$. Thus, the Sun would *barely* be resolved into a disk to the naket eye, as opposed to just looking like a point.

(c) How bright it looks is flux. If F_E is the flux as seen from the earth, and F_N is the flux as seen from Neptune, then we have:

$$F_E = \frac{L_\odot}{4\pi(1\ \text{AU})^2}$$

$$F_N = \frac{L_\odot}{4\pi(30\ \text{AU})^2}$$

Divide these two to get the ratio of fluxes:

$$\frac{F_E}{F_N} = \frac{L_\odot/(4\pi(1\text{ AU})^2)}{L_\odot/(4\pi(30\text{ AU})^2)}$$

Cancel out the stuff that appears on the top and the bottom:

$$\frac{F_E}{F_N} = \frac{(30\text{ AU})^2}{(1\text{ AU})^2} = 30^2$$

$$\boxed{\tfrac{F_E}{F_N} = 900}$$

The sun appears 900 times dimmer from Neptune than it does from Earth. Note that even though this is a lot dimmer, the Sun would still be the brightest star in Neptune's sky, by far.

A few of you tried to use $L = (\sigma T^4)(4\pi R^2)$ to show this. While you end up with a 900, what you have done doesn't make sense. Even though you've got a variable with length units (R) here, it's important to know what the variables mean when you use a given equation. This equation here tells you the luminosity of an object when you know the temperature and radius of that object. We aren't given the radius of the Earth or Neptune, we're given the distance between the Sun and each of those planets. What's more, we're not trying to figure out the Luminosity of either planet, which is what this equation would give you.

5. *Chapter 4, Question 14 in the text: On a dark night, you notice that a distant lightbulb happens to be the same brightness as a firefly that is 5 m away from you. If the lightbub is a million times more luminous than the firefly, how far away is the lightbulb?*

Use the subscript LB to describe quantities of the lightbulb, and the subscript FF for the firefly. We know:

$$F_{LB} = \frac{L_{LB}}{4\pi d_{LB}^2}$$

$$F_{FF} = \frac{L_{FF}}{4\pi(5\text{ m})^2}$$

We also know:

$$L_{LB} = 10^6 L_{FF}$$

Finally, we're told that the firefly and the lightbulb look the same brightness:

$$F_{LB} = F_{LB}$$

Substitute the first two equations into this:

$$\frac{L_{LB}}{4\pi d_{LB}^2} = \frac{L_{FF}}{4\pi(5\text{ m})^2}$$

Substitute in for L_{LB}:

$$\frac{10^6 L_{FF}}{4\pi d_{LB}^2} = \frac{L_{FF}}{4\pi(5\text{ m})^2}$$

Multiply both sides by 4π and divide both sides by L_{FF}:

$$\frac{10^6}{d_{LB}^2} = \frac{1}{(5\ \mathrm{m})^2}$$

Solve this for d_{LB}:

$$d_{LB}^2 = (5\ \mathrm{m})^2(10^6)$$

$$d_{LB} = \sqrt{25 \times 10^6\ \mathrm{m}^2}$$

$$\boxed{d_{LB} = 5{,}000\ \mathrm{m}}$$

6. *Chapter 4, Question 16 in the text: The average temperature of Earth's surface is approximately 290K. If the Sun were to become 5% more luminous (1.05 times its present luminosity), how would Earth's temperature increase?*

The equation we can use to figure out the average surface temperature of a planet is the thermal equilibrum equation, which equations how much energy the planet picks up from the Sun to how much energy it radiates:

$$\left(\frac{L_\odot}{4\pi d^2}\right)(\pi R_E^2)(1-a) = (\sigma T^4)(4\pi R_E^2)$$

where a is the Earth's albedo, R_E is the Earth's radius, d is the distance from the Sun to the Earth, and T is the Earth's temperature. As we saw in class, if you look up these values and plug them in, you get the *wrong* value (i.e. not 290K) for the average surface temperature of the Earth! As such, those of you who plugged in numbers ended up with a non-sensical answer (i.e. Earth got colder as the Sun got brighter). The reaosn is that this equation has not taken into account the Greenhouse Effect. What we shall do is assume that the Greenhouse Effect is the same both before and after the Sun got brighter. We can either have an "effective" solar luminosity that is higher to account for the Greenhouse Effect, or we can multiply by a mysterious Greenhouse Factor (call it G) whose value we don't know but which makes things work.

What we want to do is compare the temperature to the new one we get when the Sun has a new luminosity $L_{\mathrm{new}} = 1.05 L_\odot$:

$$\left(\frac{L_{\mathrm{new}}}{4\pi d^2}\right)(\pi R_E^2)(1-a) = (\sigma T_{\mathrm{new}}^4)(4\pi R_E^2)$$

(Again, maybe with a G factor out front.) As is often (though not always) the case, when trying to compare quantitites you can get a lot of mileage out of dividing two equations relating old and new quantities:

$$\frac{\left(\frac{L_\odot}{4\pi d^2}\right)(\pi R_E^2)(1-a)}{\left(\frac{L_{\mathrm{new}}}{4\pi d^2}\right)(\pi R_E^2)(1-a)} = \frac{(\sigma T^4)(4\pi R_E^2)}{(\sigma T_{\mathrm{new}}^4)(4\pi R_E^2)}$$

Notice that an awful lot of the stuff in this equation cancels out— including $L_\odot$, once you realize $L_{\mathrm{new}} = 1.05 L_\odot$, which is why you never needed to look that up.

$$\frac{1}{1.05} = \frac{T^4}{T_{\mathrm{new}}^4}$$

$$\frac{T_{\mathrm{new}}}{T} = (1.05)^{\frac{1}{4}} = 1.012$$

$$T_{\text{new}} = 1.012\ T = (1.012)(290\text{K}) = 294\text{K}$$

So the average surface temperature of the Earth would get about 4 degrees warmer. (A difference of 4 degrees Kelvin is the same as a difference of 4 degrees Centigrate. This difference would correspond to about six and a half degrees Farenheit.)