Astronomy 102, Spring 2003
Homework Set 3 Solutions

1. *1. How many radio photons (assume a wavelength of 1m) does it take to equal the energy of a single ultraviolet photon (assume a wavelength of 100nm, remembering that one nanometer (nm) is 10^{-9}m).*

$$\text{photon energy} = \frac{hc}{\lambda}$$

where h is Planck's constant (6.626, c is the speed of light, and λ is the wavelength. Thus we have:

$$E_{\text{UV}} = \frac{hc}{10^{-7}\text{ m}}$$

$$E_{\text{Radio}} = \frac{hc}{1\text{ m}}$$

At this point, we could look up and plug in values of h and c to get these two energy values. Then, we'd divide E_{UV} by E_{Radio} to figure out how many Radio photons it takes to equal the energy of one UV photon. This is equivalent to solving the equation:

$$nE_{\text{Radio}} = E_{\text{UV}}$$

for n. Note, however, that we don't actually ever need to look up h and c, if we just solve it algebraically:

$$\frac{E_{\text{UV}}}{E_{\text{Radio}}} = \frac{hc/10^{-7}\text{ m}}{hc/1\text{ m}}$$

$$= \frac{1/10^{-7}}{1/1} = \frac{10^7}{1} = 10^7$$

The h and c cancel (divide out). It takes $\boxed{\text{10 million}}$ 1m radio photons to equal the energy of a single 100nm UV photon.

Note that it was possible to get the right answer through wrong reasoning. If you just divided the wavelengths without indicating why dividing the radio wavelength by the UV wavelength is equivalent to dividing the UV energy by the radio energy, it wasn't clear you understood the problem, so I took some points off.

2. *Consider two stars, one of spectral type B and one of spectral type K. The B-star has four times the luminosity, but is twice as far away as the K-star. An astronomer observes these two stars through two filters, one centered 5000 Å, the other centered around 6500 Å.*

(a) *What can you say about the ratio of the flux of the B-star to the flux of the K-star as observed through the 5000 Angstrom filter? (I.e. is it greater than one, equal to one, or less than one.)*

(b) *What can you say about the ratio of the flux of the B-star to the flux of the K-star as observed through the 6500 Angstrom filter?*

If the discussion of the answer to this question confuses you, feel free to skip it.

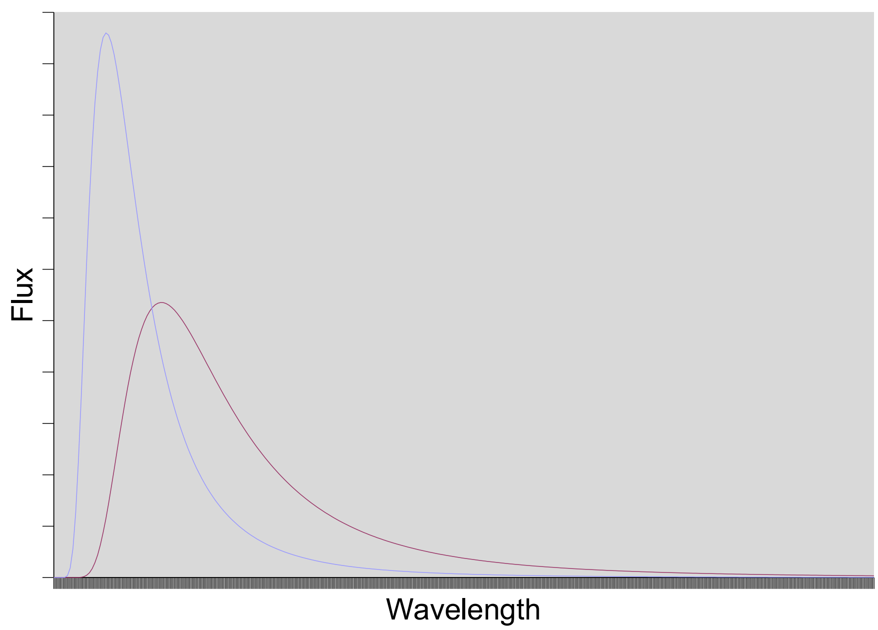
NOTE: I have to apologize for this problem: it was not possible as written, as you didn't have enough information. For those of you who didn't get the message, I did give up to two points of extra credit if you got a couple of key points. (For those of you who did get the message and didn't get the extra credit, don't worry about it: it's only two points on homework, which won't matter much to your total course grade.)

The first point (which garnered one point of extra credit) was to realize that the *total* flux of the two stars is the same. One star is four times as luminous as the other, but it's also twice as far away. Remember that twice as far away means one quarter as bright, so therefore the four times more luminous cancels out with the twice as far away. Mathematically:

$$L_B = 4L_K$$
$$d_B = 2d_K$$

$$F_B = \frac{L_B}{4\pi\ {d_B}^2} = \frac{4L_K}{4\pi\ (2d_K)^2}$$
$$= \frac{4L_K}{4\pi\ 4\ {d_K}^2} = \frac{L_K}{4\pi\ {d_K}^2} = F_K$$

Given that the *total* flux is the same, but that one star is bluer than the other (B stars are bluer than K stars), there will be some (shorter) wavelength where the B star has greater flux *at that wavelength* than the K star, and some (longer) wavelength where the K star has greater flux than the B star *at that wavelength*. This is necessary for the two to have the same total flux overall. The problem is that from the information given, it's not clear that the "crossover point" is in between the two filters requested, and thus the problem was impossible. If you were to plot the flux versus the wavelength for the two stars (I don't expect this to be obvious to you), this is what it would look like:



The *total* flux for each star is the same, so the bluer star peaks at a lower wavelength than the redder star. The only question remaining is whether that crossover is between 5000 Åand 6500 Å. (The answer to that question is left as an exercise for the two or three Physics majors in the class who know the equation for blackbody radiation.)

If you did clearly indicate that at the same *total* flux, the redder star must have more flux at some redder wavelength, and the bluer star must have more flux at some bluer wavelength, I gave you a second point of extra credit.

(Note that if you had two stars of the same size and distance, the bluer star would have more flux *at every wavelength* than the redder star. The difference here is that we don't know if the stars are the same size (they probably aren't), and they aren't at the same distance, so the red star in a sense is given an advantage.)

3. Chapter 12, Question 5: *Albiero, a star in the constellation Cygnus, is a binary system whose components are easily separated in a small amateur telescope. Viewers describe the brighter star as "golden" and the fainter one as "sapphire blue"*

(a) What does this tell you about the relative temperatures of the two stars.

(b) What does it tell you about their respective sizes?

(a) The bluer star is hotter. (You don't need to say anything about distance, relative size, luminosity, brightness, or anything else. If something is radiating thermally— as stars do— then bluer in color is higher temperature.)

(b) The first point to note is that this is a binary system, so the two stars are close to each other. Thus, they are at approximately the same distance away from the Earth as each other, or $d_G = d_S$ (G="golden", S="sapphire"). Thus, that the Golden star is brighter must also mean that the Golden star is more luminous. (The only way to make something which is less luminous appear brighter is for it to be closer.) For two objects *of the same size*, a hotter one will be more luminous, but here the cooler one is more luminous; thus, the Golden star must be larger (have a higher radius) than the Sapphire star.

Note: a few of you commented on mass, noting that for main sequence stars, more massive stars are bluer, and then concluded that the Sapphire star must be bigger because it's bluer. However, there is no reason to suspect that both of these stars are main sequence stars! You can *only* draw the more massive = bluer = more luminous comparison for main sequence stars. For arbitrary stars, you need more information (e.g. sizes, luminosities, distances, etc.) to draw comparisons. A few of you got the idea right, and then went on to conclude specific things about the nature of one or the other star, assuming that one must be a main sequence star. This was over-interpreting the information available: you don't have enough information given in the problem to determine which, if either, star is main sequence, and hence which, if either, must be another sort of star.

If the description two paragraphs up satisfies you, stop reading now. If you prefer to think in equations rather than words, you could state the same thing, using F_G and F_S for the flux (brightness) of the two stars, d_G and d_S for the distance from us to the two stars, L_G and L_S for the luminosities of the two stars, T_G and T_S for the temperatures of the two stars, and R_G and R_S for the radius (size) of the two stars.

$$F_G > F_S$$

$$\frac{L_G}{4\pi {d_G}^2} > \frac{L_S}{4\pi {d_S}^2}$$

$$d_G = d_S$$

therefore:

$$L_G > L_S$$

We also know:

$$T_S > T_G$$

The luminosity of a star at temperature T and of radius R is given by $L = (4\pi R^2)(\sigma T^4)$, thus:

$$L_G = (4\pi {R_G}^2)(\sigma {T_G}^4)$$

$$L_S = (4\pi {R_S}^2)(\sigma {T_S}^4)$$

$$(4\pi {R_G}^2)(\sigma {T_G}^4) > (4\pi {R_S}^2)(\sigma {T_S}^4)$$

The only way to satisfy this inequality given that $T_S > T_G$, and that σ is a constant (and 4 and π are just numbers), is for $R_G > R_S$.

4. Chapter 12, Question 15: *Proxima Centauri, the star nearest to Earth other than the Sun, has a parallax of 0.772 arcseconds. How long does it take light to reach us from Proxima Centauri?*

$$d = \frac{1}{p}$$

where d is the distance measured in parsecs and p is the parallax measured in arcseconds. (Note! You must be careful here. This equation only works if the quantities are in the right units. A few of you correctly used a tangent, but then took the tangent of 0.772 *radians*; 0.772 radians is *very* different from $0.772''$.) Thus, we can get:

$$d = \frac{1}{0.772}\text{pc} = 1.2953 \text{ pc}$$

(To those of you that object that I've kept two too many significant figures, remember that it's important to keep an extra digit or two for *intermediate* numbers, and only cut down to the proper number of significant figures for your final result.)

Next, we have to convert this to time. One way to do this is to look up the conversion from parsecs to meters (or kilometers), and use our knowledge of the speed of light to figure out how long it will take light to travel that distance. That works, and is fine (although a few of you got in trouble by not using enough precision on the conversion between parsecs and km).

There's a faster way, though, which is to remember that one parsec is 3.26 light-years, and realize that the speed of light is also one light-year per year. (That's how a light-year is defined: it is the distance that light travels in one year.)

Thus, we have:

$$d = 1.2953 \text{ pc} \left(\frac{3.26 \text{ light} - \text{years}}{1 \text{ pc}}\right) = 4.22 \text{ light} - \text{years}$$

Therefore, it will take $\boxed{\text{4.22 years}}$ for the light from Proxima Centauri to reach us.

5. Chapter 13, Questions 11 and 12: ***11.*** *The Sun shines by converting mass into energy according to Einstein's well-known relationship $E = mc^2$. Show that if the Sun produces 3.78×10^{26} J of energy per second it must expend 4.2 million metric tons (4.2×10^9) of mass per second.*

12. *Assume the Sun has been producing energy at a constant rate over its lifetime of 4.5 billion years (1.4×10^{17} s).*

(a) *How much mass has it lost creating energy over its lifetime?*

(b) *The present mass of the Sun is 2×10^{30} kg. What fraction of its present mass has been converted into energy over the lifetime of the Sun?*

11:

$$E = mc^2$$
$$m = \frac{E}{c^2}$$

Thus the amount of mass converted in one second is:

$$\frac{3.78 \times 10^{26} \text{ J}}{(3.00 \times 10^8 \text{ m s}^{-1})^2} = 4.2 \times 10^9 \text{ kg}$$

Whee!

12: (a)

$$(4.2 \times 10^9 \ \frac{\text{kg}}{\text{s}})(1.4 \times 10^{17} \text{ s})$$
$$= 5.9 \times 10^{26} \text{ kg}$$

(b) A total of 5.9×10^26 kg have been converted. The fraction this is of the total sun's mass is:

$$\frac{5.9 \times 10^{26}\ \mathrm{kg}}{2 \times 10^30\ \mathrm{kg}} = 3 \times 10^{-4}$$

Thus $\boxed{0.0003 \text{ or } 0.03\%}$ of the Sun's mass has been converted to energy over the last four and a half billion years.

6. *Consider a two large clouds of gas with solar abundances (i.e. mostly Hydrogen and Helium, with just a little bit of other things mixed in). Take one cloud of this gas, and make sun-like stars out of it. Let the sun-like stars live their main sequence lifetimes, and then (somehow, magically) disperse those sun like stars back into a cloud of gas. Qualitatively, how will the relative elemental abundances in the cloud which was processed through sun-like stars compare to the cloud which just remained a large, quiet cloud of gas?*

The key point here is that while the gas is inside stars, the stars are performing fusion. This produces energy, and it does slightly (see previous problem) decrease the total mass of the gas. Most significantly for relative abundances of elements, however, is that Hydrogen is being converted into Helium.

Therefore, gas which has been through a generation of stars will have a greater fraction of Helium compared to Hydrogen than gas which has not.

(*That's the end of the solution; the rest is commentary.*)

That was as much as you could say given the problem and given what we've discussed in class. A number of you indicated that the cloud which has been through stars will also have a greater abundance of heavier elements, which is correct (although I didn't require you to state this to get full credit on the problem).

A lot of you overstated the situation. I didn't take off for this if you got the basic point that the gas which has gone through stars will have more Helium. Some of you, however, stated that *all* of the Hydrogen would be converted to Helium. This is not correct; as I mentioned in class, at its current luminosity, the Sun has enough Hydrogen to keep shining for something like 80 billion years, which is much longer than it will shine. Stars do not tend to convert *all* their Hydrogen to Helium, since fusion only happens at the core (and sometimes near the core, especially for more massive stars, as we'll discuss in the next week or two). Even stating that the cloud which has been through stars will have "much" more Helium or "far" more Helium is probably overstating the situation.

Note that this was a toy model. In reality, there are two things that cloud this picture. First, when making stars out of gas clouds, only a fraction of the mass of the gas gets put into stars; the rest gets blown away or stays as gas. Second, when stars are done with their lives, they *do* tend to throw a lot of their mass back out into the interstellar medium, but not all of it: some of the mass of the star is left behind as some sort of stellar remnant. (We'll talk about those much more in coming weeks.) However, the basic idea is right: gas clouds all start out with the same "primordial" abundance of elements. As they go through multiple generations of stars, they get enriched in heavier elements. Thus, by looking at the relative abundance of heavy elements and Hydrogen, you can figure out something about how many times a gas cloud has been formed into stars which later ended their lives, putting heavy-element-enriched material back into the interstellar medium.

Even though we haven't talked about the ISM to this level of detail yet, the point of this problem was that *all* you need to know is that stars are doing fusion in order to figure out that processing gas through stars increases the relative abundance of elements heavier than Hydrogen.