Astronomy 102: Stars and Galaxies

FINAL EXAM REVIEW SOLUTIONS

1. **(a)** 1/16 is $(1/2) \times (1/2) \times (1/2) \times (1/2)$, so just under four half-lives have elapsed. This would make it about 4×1.26 billion years in the future, or about $\boxed{\text{5 billion years}}$ in teh future.

(b) Probably a big ol' red giant. The Sun is about halfway through it's 10-billion year lifetime, so 5 billion years from now it will just be reaching the end. I would also give you credit if you said it was a White Dwarf if you indicated that it had just reached that stage.

2. **(a)**

$$
\begin{aligned}
z &= \frac{d}{c\, t_H} \\
d &= z\, c\, t_H \\
d &= (0.158)\,(1\,\text{lyr yr}^{-1})\,(13.8 \times 10^9\,\text{yr}) \\
d &= 2.18\,\text{billion lyr} \;=\; 669\,\text{Mpc}
\end{aligned}
$$

(b) If the distance is 669 Mpc now, and the Universe is 0.158 times larger now than it was then, we can use:

$$
\begin{aligned}
\frac{d_{\text{Now}}}{d_{\text{Then}}} &= 1 + z \;=\; 1.158 \\
d_{\text{Then}} &= \frac{d_{\text{Now}}}{1+z} \\
d_{\text{Then}} &= \frac{669\,\text{Mpc}}{1.158} \;=\; 558\,\text{pc}
\end{aligned}
$$

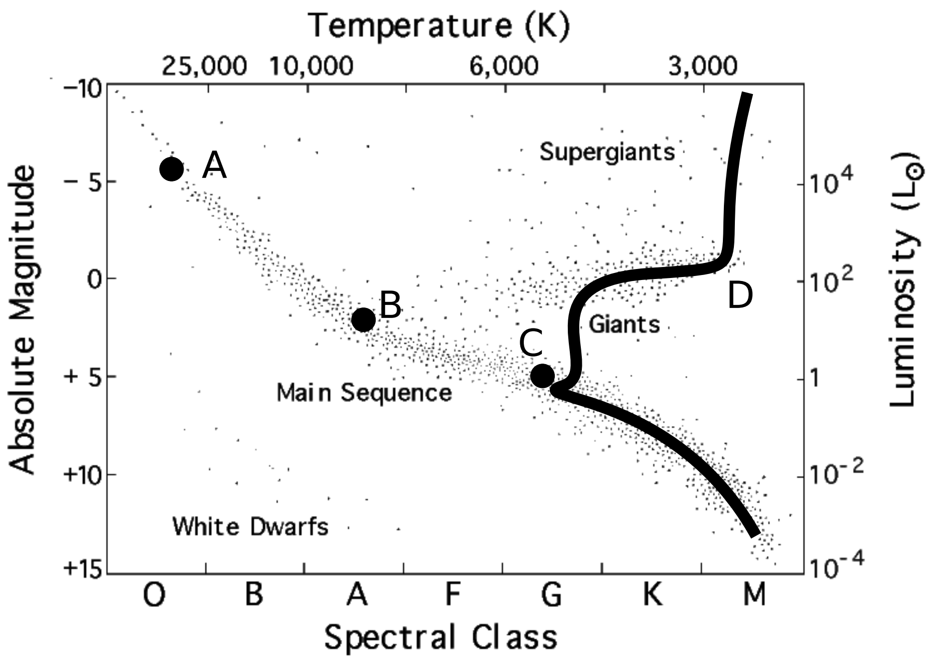
In fact, the d in the Hubble Law equation is the distance the light travelled, which is between d_{Then} and d_{Now}, but the relative *fractional* difference you calculate between d_{Then} and d_{Now} in this problem is accurate.

(c) Almost certainly not. The light took more than 2 billion years to reach us, which is more than 20 times the "typical" quasar lifetime. 3C273 is probably not a quasar any more; we see it as a quasar because we're seeing it from an earlier phase of its life when it still was a quasar.

3. **(a)** No. The Cosmological principle only applies on the largest scales — scales of hundreds of millions, or even billions, of lightyears. If you look at one region of the Universe that includes a bunch of galaxy clusters, it looks pretty much the same (same density, same total amount and colors of light, same kinds of structures) as another region of the Universe.

(b) Dark Matter is what holds galaxies together, so that is the one that clumps into galaxies, but is pretty smoothly distributed within galaxies. (Think of it as like water in lakes.) Dark Energy's effects are only seen on the very largest of scales — the acceleration of the Universe's expansion. Because we don't see the effects on smaller scales, we can be pretty sure that Dark Energy is spread out very uniformly throughout the Universe. Normal Matter, on the other hand, clumps a lot. The densenst clumps of Normal Matter tend to be stars and planets; we live on (as the density of the Universe goes) an *extremely* dense clump of Normal Matter. Even the air on Earth is *much* higher density than the average density of stuff in the Universe.

4. (a–d)



(e) For a cluster of stars that form all at once, at any time later there will be a point along the main sequence were *hotter, bluer* stars have lifetimes shorter than the age of the cluster. Those stars will no longer be on the main sequence; if they are still giants (i.e. they left the main sequence recently), they will be off to the right on the diagram somewhere. The point which is the boundry between stars that haven't left the main sequence and stars that have is at a single mass; that point is the main sequence turnoff. If you know how long a star of a given mass lives (e.g. from the theory of stellar evolution), by seeing what mass the main sequence turnoff of a given cluster corresponds to, you can figure out the age of the cluster.

5. Some— indeed, most— of the low-mass stars haven't had time to die yet. The Sun will live about 10 billion years. Stars which are less massive than the Sun will live longer. You don't have to go down very far in mass before you find stars that will live for more than 13.7 billion years, the age of the Universe. So, no matter how soon those stars formed, none of them have become white dwarfs yet. Meanwhile, *most* of the massive stars that have formed have become neutron stars or black holes; the only ones that haven't are the ones that have formed very recently in the history of the Universe.

6. (a)

$$\frac{d}{1\,\mathrm{pc}} = \frac{1''}{p}$$
$$d = \frac{1}{0.38}\,\mathrm{pc}$$
$$d = 2.63\,\mathrm{pc}$$

(b) Brightness goes as one over distance squared, so if the other star is 25 times dimmer, it must be $\sqrt{25} = 5$ times farther away.

$$d_{\mathrm{other}} = (5)\,(2.63\,\mathrm{pc}) = 13.2\,\mathrm{pc}$$

(c) For a star at 13.2 pc, you'd expect to measure parallax:

$$\frac{p}{1''} = \frac{1\,\mathrm{pc}}{d}$$
$$p = \frac{1}{13.2}'' = 0.076$$

In fact, we measure a parallax that's a lot smaller than that, which means the star is a lot (about ten times) farther away than we thought it was from its brightness. This star must be a whole lot more luminous than Sirius – it probably is larger in radius. Of, if you accept that the star and Sirius really are exactly the same from the spectrum, then there would have to be dust between us and *Sirius*, making Sirius dimmer than it ought to be. (In reality, there's not that much dust between us and Sirius.)

7. (a) The two stars are at the same distance away from us, so the brightness ratio is the same as the luminosity ratio:

$$\frac{B_W}{B_R} = \frac{L_W}{L_R} = \frac{4\pi\, {R_W}^2\,\sigma\, {T_W}^4}{4\pi\, {R_R}^2\,\sigma\, {T_R}^4}$$
$$\frac{B_W}{B_R} = \left(\frac{R_W}{R_R}\right)^2 \left(\frac{T_W}{T_R}\right)^4$$
$$\frac{B_W}{B_R} = \left(\frac{6378\,\mathrm{km}}{100 \times 695500\,\mathrm{km}}\right)^2 \left(\frac{6000\,\mathrm{K}}{3000\,\mathrm{K}}\right)^4$$
$$\frac{B_W}{B_R} = 1.35 \times 10^{-7}$$

The white dwarf is a *lot* dimmer than the red giant.

(b) Probably not. If the two stars formed together, then when they formed the star that would become the white dwarf had *greater mass* than the star that would become the red giant. A white dwarf is more evolved than a red giant, and more massive stars go through their lives faster. However, it's complicated, because a lot of mass is lost when a star becomes a white dwarf; some mass is lost to stellar winds in the red giant phase, and a bunch of mass is lost when a planetary nebula is ejected. So, even though the star that is now the white dwarf had to be more massive when the two stars started, right now it is possible that the two stars have the same mass, as the white dwarf has lost a greater fraction of their mass. That would be some nice fine-tuning, though.

(c) We're looking for another white dwarf that is $1/1.35 \times 10^{-7}$ brighter:

$$\frac{B_{\text{other}}}{B_W} = \frac{1}{1.35 \times 10^{-7}}$$

However, it's an identical white dwarf, so $L_{\text{other}} = L_W$. Thus:

$$\begin{aligned}
\frac{B_{\text{other}}}{B_W} &= \frac{\frac{L_{\text{other}}}{4\pi\, d_{\text{other}}{}^2}}{\frac{L_W}{4\pi\, d_W{}^2}} \\
\frac{B_{\text{other}}}{B_W} &= \left(\frac{d_W}{d_{\text{other}}}\right)^2 \\
d_{\text{other}} &= d_W \sqrt{\frac{B_W}{B_{\text{other}}}} \\
d_{\text{other}} &= (200\,\text{pc})\,\sqrt{1.35 \times 10^{-7}} \\
d_{\text{other}} &= 0.073\,\text{pc}
\end{aligned}$$

We don't know of any stars this close— almost certainly there is no white dwarf that close! The closest star we know is about 1.3 pc away.

8. **(a)** The yellow one is lower temperature than the blue one; color correlates directly to temperature. As a binary system, the two stars are far away, so the brighter one must be higher luminosity: the yellow one has a higher luminosity than the blue one. However, the blue one would have higher luminosity if they were the same size, so the yellow one must be bigger.

(b) There are two possibilities. First, it's possible that the blue star is a main sequence star. The yellow star would then be a giant or supergiant, a more massive star that is further along through its evolution.

The second possibility is that the yellow star is a main sequence star, and the blue star is a white dwarf (that recently became a white dwarf). In this case, the blue star would have started out as the more massive star, since in this case the blue star is further along through its evolution.

Both of these cases have the yellow star with a larger radius than the blue star.

9. Some of these take some calculations to figure out where to plot them.

(b) What's the luminosity of this star? Well, from the parallax, we know the distance is 1/0.1=10 pc. We can put this together with the brightness to find the luminosity:

$$B = \frac{L}{4\pi\, d^2}$$
$$L = 4\pi\, B\, d^2$$
$$\frac{L_*}{L_\odot} = \frac{4\pi\, B_*\, {d_*}^2}{4\pi\, B_\odot\, {d_\odot}^2}$$
$$\frac{L_*}{L_\odot} = \left(\frac{B_*}{B_\odot}\right)\left(\frac{d_*}{d_\odot}\right)^2$$
$$\frac{L_*}{L_\odot} = (2.4\times 10^{-12})\left(\frac{10\,\mathrm{pc}}{1\,\mathrm{AU}}\right)^2\left(\frac{206265\,\mathrm{AU}}{1\,\mathrm{pc}}\right)^2$$

The last factor there is to convert the Units; the end thing should be unitless ,since this is just a ratio of luminosities we're talking about.

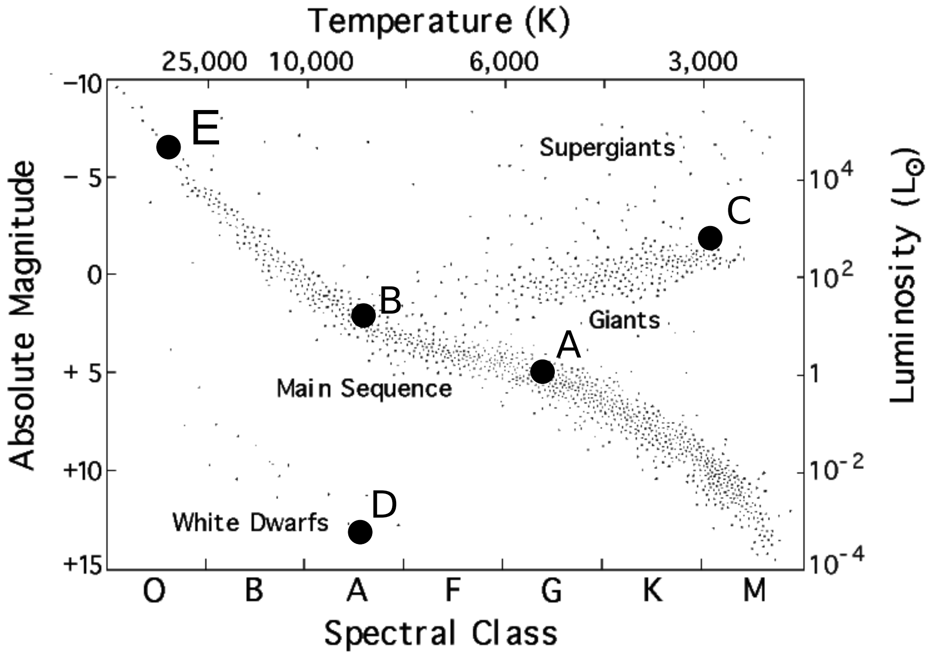
$$\frac{L_*}{L_\odot} = 10$$

(c) Now we don't know distances, but we know radii, so we can still find $L_*/L_\odot$:

$$\frac{L_*}{L_\odot} = \frac{4\pi\, {R_*}^2\,\sigma\, {T_*}^4}{4\pi\, {R_\odot}^2\,\sigma\, {T_\odot}^4}$$
$$\frac{L_*}{L_\odot} = \left(\frac{R_*}{R_\odot}\right)^2\left(\frac{T_*}{T_\odot}\right)^4$$
$$\frac{L_*}{L_\odot} = (80)^2\left(\frac{3,000}{5,780}\right)^4$$
$$\frac{L_*}{L_\odot} = 460$$

(d) This is just like the last problem:

$$\frac{L_*}{L_\odot} = \left(\frac{R_*}{R_\odot}\right)^2\left(\frac{T_*}{T_\odot}\right)^4$$
$$\frac{L_*}{L_\odot} = \left(\frac{6,378\,\mathrm{km}}{695,500\,\mathrm{km}}\right)^2\left(\frac{9,000}{5,780}\right)^4$$
$$\frac{L_*}{L_\odot} = 4.9\times 10^{-4}$$


Temperature (K)
25,000
10,000
6,000
3,000
−10
− 5
0
+ 5
+10
+15
Absolute Magnitude
E
Supergiants
C
B
Giants
A
Main Sequence
D
White Dwarfs
10^4
10^2
1
10^{-2}
10^{-4}
Luminosity ($L_\odot$)
O
B
A
F
G
K
M
Spectral Class

zx