Astronomy 102, Summer 2006

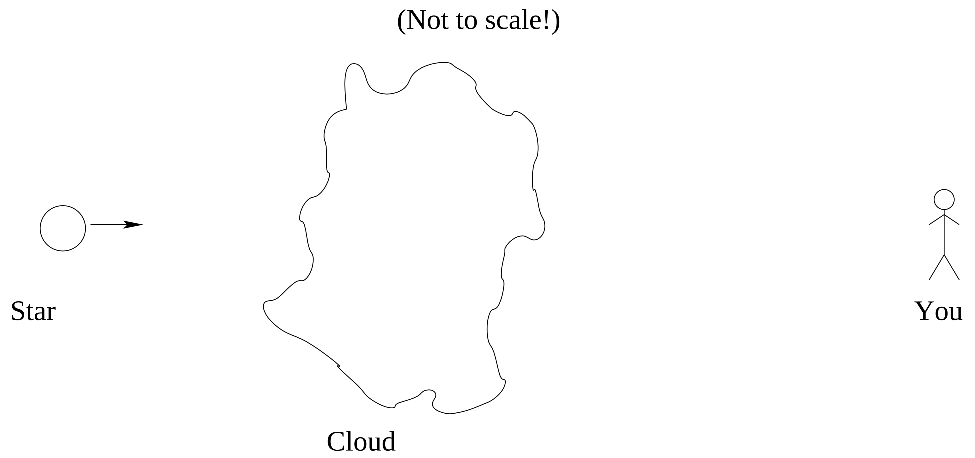
Review Exam 1 Solutions

1. Not much. If the Potassium-40/Argon-40 ratio is *higher*, it means that the rock is *younger* — it solidifed more recently. In this case, you could work out if you used the equations given in class, or if you looked at the plot shown in class,that these rocks are about 3 billion years old. However, this is not a "discrepancy" that should cast doubt on the age of the Solar System. It is the age of the oldest rocks that give us the age of the solar system. Some rocks may well have solidifed after the formation of the Solar System. Geological processes on planets can liquify material, which then re-solidifes, and which may get blasted off of the planet by an impact with an asteroid.

2. The rate of decays is proportional to the amount you have. Each given atom has a certain probability of decaying in the next second. The more dice you roll, the more 1's you get, even though you can't predict the roll of any one die. After one half-life, half of the original sample is left, so the decay rate will be half the original rate. After two half lives, only a quarter (1/2 times 1/2) of the original sample is left, so the decay rate will be a quarter the original decay rate, or $\boxed{\text{5 decays per second}}$.

3. If the intervening cloud is cold and dark, it can't be responsible for the emission lines. (Emission lines would make it not dark, and in any event you must excite a cloud somehow to make it emit emission lines, which would also make it unlikely to be cold— at least, if the emission lines we're talking about are optical emission lines.) Therefore, the star must be emitting the emission lines, which is unusal; most stars show absorption lines, not emission lines.

The intervening cloud is absorbing the light. To explain the difference in Doppler shift, the intervening cloud must be at rest, while the emission-line star is approaching you.



4. **(a)**

$$\lambda f = c$$
$$f = \frac{c}{\lambda}$$
$$f = \frac{3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}}{6563 \times 10^{-10}\,\mathrm{m}}$$
$$\boxed{f = 4.57 \times 10^{14}\,\mathrm{s^{-1}}}$$

(b)

$$E = h\,f$$
$$E = (6.626 \times 10^{-34}\,\mathrm{J\,s})(4.57 \times 10^{14}\,\mathrm{s^{-1}}$$
$$\boxed{E = 3.03 \times 10^{-19}\,\mathrm{J}}$$

(c)

$$(120\,\mathrm{J})\left(\frac{1\,\mathrm{photon}}{3.03 \times 10^{-19}\,\mathrm{J}}\right)$$
$$\boxed{3.96 \times 10^{20}\,\mathrm{photons}}$$

(d)

$$\frac{\lambda_{\mathrm{obs}} - \lambda_{\mathrm{emit}}}{\lambda_{\mathrm{emit}}} = \frac{v}{c}$$

In this case, λ_{emit} is 6563Å. We need to find λ_{obs}:

$$\lambda_{\mathrm{obs}} - \lambda_{\mathrm{emit}} = \left(\frac{v}{c}\right)(\lambda_{\mathrm{emit}})$$
$$\lambda_{\mathrm{obs}} = \lambda_{\mathrm{emit}}\left(1 + \frac{v}{c}\right)$$

We know c in m/s, not mph, so we have to convert:

$$c = 3.00 \times 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}\left(\frac{1\,\mathrm{mi}}{1{,}609\,\mathrm{m}}\right)\left(\frac{3{,}600\,\mathrm{s}}{1\,\mathrm{h}}\right)$$
$$c = 6.70 \times 10^{8}\,\mathrm{mph}$$

Now we can figure out the Doppler shifted wavelength. Note that the velocity we have is *negative* in this equation, because the ball is *approaching* us. We should get a blueshift, or a wavelength which is less than 6563Å.

$$\lambda_{\mathrm{obs}} = (6563\,\text{Å})\left(1 + \frac{-90\,\mathrm{mph}}{6.70 \times 10^{8}\,\mathrm{mph}}\right)$$
$$\boxed{\lambda_{\mathrm{obs}} = 6563\text{Å}}$$

Is that a blueshift? Hard to say to this precision! 90mph is so bloody much less than the speed of light that it's very difficult to see the blueshfit at all.

5. The brightest star in the sky during the day is, of course, the Sun....

(a) They are at the same distance, so the ratio of brightnesses will be the same as the ratio of luminosities:

$$\frac{L_A}{L_B} = \frac{4\pi \, {R_A}^2 \, \sigma \, {T_A}^4}{4\pi \, {R_B}^2 \, \sigma \, {T_B}^4}$$

The factors of 4π and σ appear on both the top and bottom, so we can cancel them out:

$$\frac{L_A}{L_B} = \left(\frac{R_A}{R_B}\right)^2 \left(\frac{T_A}{T_B}\right)^4$$

Solve this for the ratio of radii:

$$\left(\frac{R_A}{R_B}\right)^2 = \left(\frac{L_A}{L_B}\right) \left(\frac{T_B}{T_A}\right)^4$$

$$\frac{R_A}{R_B} = \left(\frac{L_A}{L_B}\right)^{1/2} \left(\frac{T_B}{T_A}\right)^2$$

Remember that something to the 1/2 is just the square root of that something:

$$\frac{R_A}{R_B} = \sqrt{\left(\frac{L_A}{L_B}\right)} \left(\frac{T_B}{T_A}\right)^2$$

Now we can put in numbers:

$$\frac{R_A}{R_B} = \sqrt{9,200} \left(\frac{30,000\,\mathrm{K}}{10,000\,\mathrm{K}}\right)^2$$

$$\frac{R_A}{R_B} = 300$$

Sirius A is a lot bigger than Sirius B. (In fact, Sirius B is what is called a "white dwarf" star, and its size is much more comparable to the size of the Earth than to the size of the Sun.)

(b) Same sort of thing, only it's brightness ratios we care about. Let's divide equations again:

$$\frac{B_A}{B_\odot} = \frac{\frac{L_A}{4\pi \, {d_A}^2}}{\frac{L_\odot}{4\pi \, {d_\odot}^2}}$$

Cancel what is on the top and bottom, and rearrange to get rid of the over-over stuff:

$$\frac{B_A}{B_\odot} = \left(\frac{L_A}{L_\odot}\right) \left(\frac{d_\odot}{d_A}\right)^2$$

The distance from the Earth to the Sun is $d_\odot = 1$ AU. Put in:

$$\frac{B_A}{B_\odot} = (25) \left(\frac{1\,\mathrm{AU}}{8.6\,\mathrm{lyr}}\right)^2$$

Well, hmm, the units didn't all cancel out. The ratio of distances is just a ratio (a distance divided by a distance), so we should be able to make the units go away. But we have to convert AU to light-years for that to happen.

$$\frac{B_A}{B_\odot} = (25)\left(\frac{1\,\mathrm{AU}}{8.6\,\mathrm{lyr}}\right)^2 \left(\frac{1\,\mathrm{pc}}{206,265\,\mathrm{AU}}\right)^2 \left(\frac{3.26\,\mathrm{lyr}}{1\,\mathrm{pc}}\right)^2$$

Notice! I had to *square* the unit conversion factors, since we had factors of AU^2 and lyr^2 to get rid of! Now all the units cancel, and we can multiply it out to get:

$$\frac{B_A}{B_\odot} = 2.6 \times 10^{-11}$$

Sirius, despite being the brightest star in the night sky, literally can't hold a candle to the Sun. (If you're holding a candle, you can read by its light. But you can't read by the light of Sirius.) (Well, OK, maybe I'm taking that "literally" thing too far.)