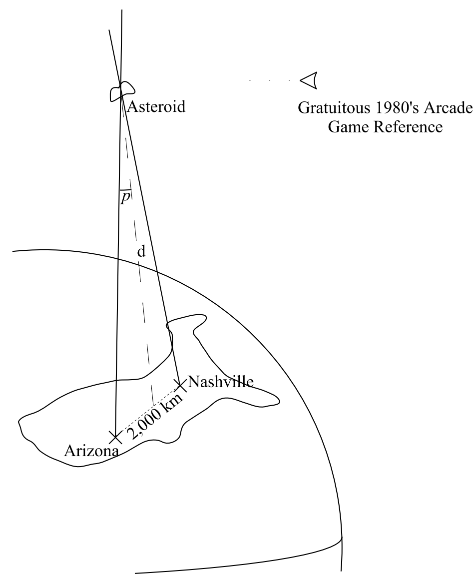
Astronomy 102, Summer 2006

Review Exam 2 Solutions

1.

(a)



(b) The angle p is small ($2''$ is a small angle), so we can use the formula:

$$p = \frac{h}{d}$$

In this case, we know h (it's 1,000 km, or half of the baseline between Nashville and Arizona), and we want to find d, so we solve this equation:

$$d = \frac{h}{p}$$

To use this equation, though, we have to convert p from arcseconds to radians:

$$p = (2'') \times \frac{1 \text{ radian}}{206,265''} = 9.7 \times 10^{-6} \text{ radians}$$

If we plug in this minimum angle we can measure, we'll get the maximum d we can measure. (Any d lower than that would give us a bigger angle, which would only be easier to measure with our precision.) This will give us d in km (the same units as we have h in).

$$d = \frac{1,000 \text{ km}}{9.7 \times 10^{-6}}$$

$$\boxed{d = 1 \times 10^{8} \text{ km}}$$

For purposes of comparison, convert this to AU:

$$d = 1.03 \times 10^{8} \text{ km} \times \left(\frac{1 \text{ AU}}{1.496 \times 10^{8} \text{ km}}\right) = \boxed{0.7 \text{ AU}}$$

(c) This is much larger than the distance from the Earth to the Moon, and is comparable to the distance from the Earth to the Sun.

(d) You will want it to be as high in the sky as it gets. If it's near rising or setting, the triangle will be "squished", giving you a smaller angle. Ideally, you want it to be just *past* its highest point in the sky as viewed from Nashville, and just *before* its highest point in the sky as viewed from Arizona, so that it's nicely positioned up between the two as drawn. Anywhere else, and you'll have to do some trig to take into account the "squished" triangle.

2. This one's a little bit hard. It's tempting to say "everyting else is the same, so the colors must be the same." While this is the right answer, it's an oversimplified explanation.

If the *angular* sizes are the same, then what you really know is that the size-to-distance ratio for the two stars is the same:

$$\frac{R_1}{d_1} = \frac{R_2}{d_2}$$

You also know the brightnesses are the same:

$$B_1 = B_2$$

So you know:

$$\frac{L_1}{4\pi \, {d_1}^2} = \frac{L_2}{4\pi \, {d_2}^2}$$

But you don't know how L_1 and L_2 compare, since you don't directly know how R_1 and R_2 compare.

What we're really after is temperatures, which relates to luminosity and size (two other things we've talked about) through:

$$L = 4\pi \, R^2 \, \sigma \, T^4$$

Solve this for temperature, since it's what we care about:

$$T^4 = \frac{L}{4\pi \, \sigma \, R^2}$$

To see how the temperatures of the two stars compare, divide their temperatures; if $T_1 > T_2$, we'll know that Star 1 is bluer. If you can't figure out how the temperatures compare, then you can't say anything about colors!

$$\left(\frac{T_1}{T_2}\right)^4 = \frac{\frac{L_1}{4\pi \, \sigma \, {R_1}^2}}{\frac{L_2}{4\pi \, \sigma \, {R_2}^2}}$$

$$\left(\frac{T_1}{T_2}\right)^4 = \left(\frac{L_1}{4\pi \, \sigma \, {R_1}^2}\right)\left(\frac{4\pi \, \sigma \, {R_2}^2}{L_2}\right)$$

$$\left(\frac{T_1}{T_2}\right)^4 = \left(\frac{L_1}{L_2}\right)\left(\frac{R_2}{R_1}\right)^2$$

Hmm. We don't know the luminosity relationship, so it would seem we're stuck. Aha! We've got another relationship between luminosity and distance above from the brightnesses being equal; we can turn that into a ratio of luminosities with a little algebra:

$$\frac{L_1}{L_2} = \frac{4\pi \, {d_1}^2}{4\pi \, {d_2}^2}$$

$$\frac{L_1}{L_2} = \left(\frac{d_1}{d_2}\right)^2$$

OK. It's not clear that that helped, since we also don't know the distances. . . but we do know something about radii and distances together, from the angular sizes, so perhaps we've made progress. Let's try putting that in our temperature ratio equation:

$$\left(\frac{T_1}{T_2}\right)^4 = \left(\frac{d_1}{d_2}\right)^2\left(\frac{R_2}{R_1}\right)^2$$

Hey! Now we're making progress:

$$\left(\frac{T_1}{T_2}\right)^4 = \left(\frac{R_2/d_2}{R_1/d_1}\right)^2$$

But we already know that $R_2/d_2 = R_1/d_1$, so now we have that $T_1/T_2 = 1$, or $T_1 = T_2$. So we know the color of the stars is the same!

Be very careful though. We do *not* know that which star, if either, is more luminous, nor do we know which star is more distant! They could be at the same distance; or, one could be a yellow supergiant, whereas the other is a yellow main sequence star that's much closer (thereby balancing the angular size from being physically smaller and the brightness from being less luminous).

If you get a problem like this and don't know immediately how to do it, don't just freeze up and write down random equations. Explain what you know and what you need to figure out. If you're thinking along the right lines, you may get some partial credit. Also, by explaining that to yourself, you may actually be able to guide yourself through figuring out what it is that you need to do.

3. (a)

$$\lambda_{\mathrm{max}} = \frac{2.9\times10^7\,\text{Å}\,\mathrm{K}}{T} = \frac{2.9\times10^7\,\text{Å}\,\mathrm{K}}{1500\,\mathrm{K}}$$

So:

$$T = \frac{2.9\times10^7\,\text{Å}\,\mathrm{K}}{\lambda_{\mathrm{max}}} = \frac{2.9\times10^7\,\text{Å}\,\mathrm{K}}{1500\,\text{Å}}$$

$$\boxed{T = 19,000\,\mathrm{K}}$$

A star of a temperature like that is an O or a B star.

(a) Same calculation, on you get $\boxed{\mathrm{T} = 3{,}700\,\mathrm{K}}$. This is a K-type star.

(c) From the parallaxes, we know:

$$d_A = \frac{1}{0.0080}\,\mathrm{pc} = 125\,\mathrm{pc}$$

$$d_B = \frac{1}{0.080}\,\mathrm{pc} = 12.5\,\mathrm{pc}$$

We also have

$$B_A = B_B$$

So we know:

$$\frac{L_A}{4\pi\,{d_A}^2} = \frac{L_B}{4\pi\,{d_B}^2}$$

Solve this for the ratio of luminosities:

$$\frac{L_A}{L_B} = \frac{4\pi\,{d_A}^2}{4\pi\,{d_B}^2}$$

$$\frac{L_A}{L_B} = \left(\frac{d_A}{d_B}\right)^2 = \left(\frac{125\,\mathrm{pc}}{12.5\,\mathrm{pc}}\right)$$

$$\boxed{L_A/L_B = 100}$$

Sanity check: B is much closer, but has the same brightness as A, so B is dimmer. Good, that matches with what we got.

(d) If Star A is a main sequence B star, then just looking at where those stars fall on the H-R diagram, we can conclude that its luminosity is a bit less than 100 $L_\odot$. Therefore, the luminosity of Star B is a bit less than 1 $L_\odot$, from problem C. Star B is a K-type star; a K-type star that's of luminosity a bit less than 1 $L_\odot$ is much closer to the main sequence than it is to the giant branch, so it's probably a $\boxed{\text{main sequence star}}$.

4.

(a)

$$d = \frac{1}{p}$$

so...

$$p = \frac{1}{d}$$

when d is in pc and p is in arcsec. We can then figure out that the parallax of the Andromeda Galaxy would be:

$$p = \frac{1}{800,000}'' = 1.25 \times 10^{-6}{}''$$

(or $1 \times 10^{-6}{}''$, one millionth of an arcsecond, to one significant figure). You would need to measure parallax at *least* this well to measure the distance to the Andromeda Galaxy, ideally a factor of five or ten better. The best we can do right now with parallax is about $10^{-3}{}''$, so this is not how we know the distance to M31.

(b) When I've given this on a test, may people noted that 30 AU is insignificant in comparison to 800,000 pc, so Neptune is not appreciably closer to M31 than Earth is. This is correct, but in so saying you are implicitly assuming a "flat Earth" model, where everything is "up". The planets are up, the stars are farther up.... The real Universe, of course, is three-dimensional, and Neptune may not even be in the same direction as M31! In fact, it may be every so very slightly farther from Neptune to M31 than from Earth; it all depends on where Neptune is in it's orbit. And, even if all three were lined up, the difference might be 31 AU rather than 29 AU. More likely, they're off at some different angle, and you'd have to do a bunch of messy trig to find out the difference in the distance. Ultimately, though, it doesn't matter, since only have the M31 distance to one significant figure, and any difference is going to be *way, way, way* less than that...so the distance from Neptune to M31 is 800,000 pc.

This does *not* mean, however, that you'd need to measure angles to the same precision! Think about how parallax works. You look at something from two points of view, and measure the difference in the direction from one point of view to the other. Neptune's orbit is 30 times the size of Earth's orbit, so there's going to be a much bigger parallax as measured from Neptune! In fact, it will be 30 times bigger, so you only need to measure angles to:

$$(30)(1.25 \times 10^{-6}) = \boxed{4 \times 10^{-5}{}''}$$

This still isn't good enough to get Andromeda (if we can only measure to a milliarcsecond), but we are doing better than we did on Earth.

(c) The disadvantage is that Neptune's orbital period is much longer than Earth's. Therefore, you'd have to wait a very long time (about 80 years) to get from one side of Neptune's orbit to another. **NOTE!** I know that this semester, we have not talked about how long it takes planets to go around the Sun, so don't worry about it if you didn't know this part of the problem.

In the past, students have suggested handing the project off between generations of a way of getting around the disadvantage. This isn't getting around the disadvantage; this is accepting it.

Note that the important thing about parallax is getting the measurements from two points of view. So why wait? If you can afford to send a space station out to Neptune, you might well be able to send out one in the opposite direction to the other side of Neptune's orbit. Make you're measurements and compare data, and you're done all at once. Or, you can do half as well if you compare measurements from Neptune with measurements made on Earth, and save yourself the expense of sending out a second space station.

5. From the parallaxes, we have distances:

$$d_A = \frac{1}{0.10}\,\text{pc} = 10\,\text{pc}$$

$$d_B = \frac{1}{0.025}\,\text{pc} = 40\,\text{pc}$$

We are also given a brightness ratio:

$$B_B = 2\,B_A$$

What we're looking for is luminosities. B is farther, but B is also brighter— so we'd better come out with B being more luminous! We expect a ratio $L_A/L_B < 1$.

We have distances, and we have brightnesses. We have distances and brightnesses, and know how to relate all that to luminosities:

$$B = \frac{L}{4\pi\,d^2}$$

Specifically, we have:

$$B_A = \frac{L_A}{4\pi\,{d_A}^2} \qquad B_B = \frac{L_B}{4\pi\,{d_B}^2}$$

What we want is a luminosity ratio, so solve each of these for luminosity:

$$L_A = 4\pi\,{d_A}^2\,B_A \qquad L_B = 4\pi\,{d_B}^2\,B_B$$

To get the ratio, divide those two puppies:

$$\frac{L_A}{L_B} = \frac{4\pi\,{d_A}^2\,B_A}{4\pi\,{d_B}^2\,B_B}$$

$$\frac{L_A}{L_B} = \left(\frac{d_A}{d_B}\right)^2\left(\frac{B_A}{B_B}\right)$$

$$\frac{L_A}{L_B} = \left(\frac{10\,\text{pc}}{40\,\text{pc}}\right)^2\left(\frac{1}{2}\right)$$

$$\boxed{\frac{L_A}{L_B} = \frac{1}{32}}$$

This does come out with B more luminous, as expected.

6. The first Cepheid variable star's distance can be determined easily from the parallax:

$$d_{\text{C1}} = \frac{1}{p} = \frac{1}{0.022}$$

$$d_{\text{C1}} = 45.45\,\text{pc}$$

(I am keeping "too many" sig figs, since I'll use this number in subsequent calculations.)

The second Cepheid variable star is dimmer, but also farther. It has the same period, though, so it must have the *same* luminosity as the first Cepheid. (This is what makes Cepheids useful!)

$$B = \frac{L}{4\pi\,d^2}$$

$$L = 4\pi\,d^2\,B$$

$$L_{\text{C1}} = L_{\text{C2}}$$

$$4\pi\,{d_{\text{C1}}}^2\,B_{\text{C1}} = 4\pi\,{d_{\text{C2}}}^2\,B_{\text{C2}}$$

$$d_{\text{C2}} = d_{\text{C1}}\sqrt{\frac{B_{\text{C1}}}{B_{\text{C2}}}}$$

$$d_{\text{C2}} = 45.45\,\text{pc}\sqrt{\frac{3.4\times10^{-11}}{1.1\times10^{-21}}}$$

$$d_{\mathrm{C2}} = 8.00 \times 10^6\,\mathrm{pc} = 8.00\,\mathrm{Mpc}$$

That is a reasonable distance for a "nearby" galaxy (galaxies tend to be a few Mpc apart).

The first Type Ia supernova is 8.00 Mpc away, of course, since it's in the same galaxy as the first Cepheid! We can do the same thing as above to figure out the ratio of distances between the two supernovae; remember that supernovae are standard candles.

$$\begin{aligned}
L_{S1} &= L_{S1} \\
4\pi\, {d_{\mathrm{S1}}}^2\, B_{\mathrm{S1}} &= 4\pi\, {d_{\mathrm{S2}}}^2\, B_{\mathrm{S2}} \\
d_{\mathrm{S2}} &= d_{\mathrm{S1}} \sqrt{\frac{B_{\mathrm{S1}}}{B_{\mathrm{S2}}}}
\end{aligned}$$

$$d_{\mathrm{S2}} = 8.00\,\mathrm{Mpc}, \sqrt{\frac{1.5 \times 10^{-15}}{1.3 \times 10^{-19}}}$$

$$d_{\mathrm{S2}} = \boxed{860\,\mathrm{Mpc}} = 8.6 \times 10^6\,\mathrm{pc}$$

7. When the cluster stars, there are *no* white dwarfs or neutron stars, because those stars are the endpoints of star lifetimes. . . and no stars have been through their lifetime first! Neutron stars are left behind from the deaths of the most massive stars, so they will start to build up first, because the most massive stars have the shortest lifetimes. After a few times ten million years, all of the massive stars will have died, and after that the number of neutron stars will remain constant. (We have the ones we've made, but no more get made.) At that point, the number of white dwarfs stars to build up, and only keeps building up as lower and lower mass stars are able to go through their lifetimes.