Astronomy 102: Stars and Galaxies

Review Examination 3 Solutions

1. We would have *underestimated* the expansion rate of the Universe. We underestimated the brightness of the supernova. That mean that we thought it was farther than it really was. That meant that we thought we were looking further back in time than we really were. Because we thought it took longer than it really did for the Universe to expand the amount indicated by the redshift, we have underestimated the rate at which the Universe is expanding.

2.

(a)

$$z = \frac{d}{c\,t_H} = \left(\frac{50\,Mpc}{(1\text{lyr}\,\text{yr}^{-1})\,(13.8\times10^{9}\,\text{yr})}\right)\left(\frac{3.26\times10^{6}\,\text{lyr}}{Mpc}\right)$$

$$\boxed{z = 0.012}$$

(With extra sig. figs, that is $z = 0.01181$, which I'll use in subsequent calculations.)

(b)

$$z = \frac{\lambda_{\text{obs}} - \lambda_{\text{emit}}}{\lambda_{\text{emit}}}$$

$$1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}}$$

$$\lambda_{obs} = (1+z)\,\lambda_{\text{emit}} = (1.01181)\,(6562.8\,\text{Å})$$

$$\boxed{\lambda_{obs} = 6640\,\text{Å}}$$

(c)

$$B_* = \frac{L_\odot}{4\pi\,{d_*}^2}$$

$$B_{\text{Vega}} = \frac{L_{\text{Vega}}}{4\pi\,{d_{\text{Vega}}}^2}$$

$$\frac{B_*}{B_{\text{Vega}}} = \frac{\frac{L_\odot}{4\pi\,{d_*}^2}}{\frac{L_{\text{Vega}}}{4\pi\,{d_{\text{Vega}}}^2}} = \left(\frac{L_\odot}{L_{\text{Vega}}}\right)\left(\frac{d_{\text{Vega}}}{d_*}\right)^2$$

$$\frac{B_*}{B_{\text{Vega}}} = \left(\frac{1\,L_\odot}{55\,L_\odot}\right)\left(\frac{7.8\,\text{pc}}{50\times10^{6}\,\text{pc}}\right)^2$$

$$\boxed{\frac{B_*}{B_{\text{Vega}}} = 4.4\times10^{-16}}$$

No prayer of picking out this individual star with any of our telescopes we have right now!

(d) This problem takes an equation from the previous problem, but rearranges it. Start with:

$$\frac{B_*}{B_{\rm Vega}} = \left(\frac{L_\odot}{L_{\rm Vega}}\right)\left(\frac{d_{\rm Vega}}{d_*}\right)^2$$

Only now we know the brightness ratio we're looking for, and want to solve for d_*:

$$d_* = d_{\rm Vega}\sqrt{\left(\frac{B_{\rm Vega}}{B_*}\right)\left(\frac{L_\odot}{L_{\rm Vega}}\right)}$$

$$d_* = (7.8\,{\rm pc})\sqrt{\left(\frac{1}{10^{-11}}\right)\left(\frac{1\,L_\odot}{55\,L_\odot}\right)}$$

$$d_* = 3.3\times10^5\,{\rm pc} = 0.3\,{\rm Mpc}$$

That doesn't even take us to the Andromeda Galaxy! When we look at individual stars in other galaxies, we're generally looking at very bright stars.

As for the redshift... well, it would be domninated by proper motion, but ignoring that:

$$z = \frac{d}{ct_{\rm H}} = \frac{3.3\times10^5\,{\rm pc}}{(1\,{\rm lyr\,yr^{-1}})\,(13.8\times10^9\,{\rm yr})}\left(\frac{3.26\,{\rm lyr}}{1\,{\rm pc}}\right)$$

$$z = 0.00008$$

3 This is the same as the last problem, only with a different luminosity ratio:

$$d = d_{\rm Vega}\sqrt{\left(\frac{B_{\rm Vega}}{B_{\rm Cepheid}}\right)\left(\frac{L_{\rm Cepheid}}{L_{\rm Vega}}\right)}$$

Why are we still messing with Vega? Because that's the brightness ratio that we know the sensitifity for.

$$d = (7.8\,{\rm pc})\sqrt{\left(\frac{1}{1\times10^{-11}}\right)\left(\frac{1200\,L_\odot}{55\,L_\odot}\right)}$$

$$\boxed{d = 1.1\times10^7\,{\rm pc} = 11\,{\rm Mpc}}$$

Again, that's close enough that proper motion is going to be important, but let's ignore that in calculating the redshift:

$$z = \frac{d}{ct_{\rm H}} = \frac{1.115\times10^7\,{\rm pc}}{(1\,{\rm lyr\,yr^{-1}})\,(13.8\times10^9\,{\rm yr})}\left(\frac{3.26\,{\rm lyr}}{1\,{\rm pc}}\right)$$

$$\boxed{z = 0.0027}$$

3. Remember that z is change in size divided by original size. Recast the Hubble law to:

$$z = \frac{\Delta h}{h} = \frac{t_{\rm LB}}{t_H}$$

as we have done many times in class. For "lookback" time, let's use 80 years. (You can think about doing this problem 80 years from now; 80 years won't make any difference in the Hubble time to the precision we've measured it!)

$$\Delta h = 1.75m \left(\frac{80\,{\rm yr}}{13.8 \times 10^9\,{\rm yr}}\right)$$

$$\Delta\ h = 1.0 \times 10^{-8}\,{\rm m}$$

That's roughly the size of a reasonable molecule....

4. (a) This is one of the old brightness/luminosity/distance things:

$$B_{\rm SN} = \frac{L_{\rm SN}}{4\pi\, {d_{\rm SN}}^2}$$

We need to compare it to Vega:

$$B_{\rm Vega} = \frac{L_{\rm Vega}}{4\pi\, {d_{\rm Vega}}^2}$$

$$\frac{B_{\rm SN}}{B_{\rm Vega}} = \frac{\frac{L_{\rm SN}}{4\pi\, {d_{\rm SN}}^2}}{\frac{L_{\rm Vega}}{4\pi\, {d_{\rm Vega}}^2}} = \left(\frac{L_{\rm SN}}{L_{\rm Vega}}\right)\left(\frac{d_{\rm Vega}}{d_{\rm SN}}\right)^2$$

Solve this for $d_{\rm SN}$:

$$d_{\rm SN} = d_{\rm Vega}\sqrt{\left(\frac{L_{\rm SN}}{L_{\rm Vega}}\right)\left(\frac{B_{\rm Vega}}{B_{\rm SN}}\right)}$$

$$d_{\rm SN} = (7.8\,{\rm pc})\sqrt{\left(\frac{5.8 \times 10^9\, L_\odot}{55\, L_\odot}\right)\left(\frac{1}{1.6 \times 10^{-7}}\right)}$$

$$\boxed{d_{\rm SN} = 2.0 \times 10^8\,{\rm pc} = 200\,{\rm Mpc}}$$

(b) We have the distance and the redshift, so we can calculate what t_H is:

$$z = \frac{d}{c\, t_H}$$

Solve:

$$t_H = \frac{d}{c\, z}$$

$$t_H = \frac{2.00 \times 10^8\,\mathrm{pc}}{(1\,\mathrm{lyr\,yr^{-1}})\,(0.062)} \left(\frac{3.26\,\mathrm{lyr}}{1\,\mathrm{pc}}\right)$$

$$t_H = 1.1 \times 10^{10}\,\mathrm{yr} = 11 \text{ billion years}$$

Notice that I keep using the speed of light as 1 light-year per year. This saves me a bunch of pain in unit conversions; I don't have to mess with going from parsecs to meters (only parsecs to light-years) or from seconds to years.

(c) This galaxy is 200 million parsecs, or 650 million light-years away. That means that the first supernova exploded 650 million years ago, so the second supernova exploded $\boxed{\text{450 million years ago}}$. We'll see it 200 million years from now. (Actually, a little longer than that, because the Universe will have expanded during that time, and the light will have to go a bit farther as a result. You could calculate the difference!)

5. (a) Right now, we have $t_H = 13.8$ billion years, which is how old the Universe would be if the expansion rate had always been constant. If the expansion rate stays the same for the next five billion years, we would say that the Unvierse would be 18.8 billion years old if the expansion rate would always be constant…so we would say that $\boxed{t_H = 18.8 \text{ billion years}}$.

(b) In fact, the expansion rate of the Universe is accelerating. As such, five billion years from now, the expansion rate of the Unvierse will be higher. A higher expansion rate corresponds to a *lower* Hubble time, so people five billion years from now will measure $\boxed{t_H < 18.8 \text{ billion years}}$.