Astro 102 Group Problems #2
Solutions

1. This is a rate problem. We have the total amount of fuel ($M_\odot$), but we don't have the rate at which the fuel is being used. Howver, we do have the rate at which the Sun is producing energy: it has a luminosity of 3.8×10^{26} W, meaning that in one second it produces 3.8×10^{26} J of energy. We can figure out how much mass the Sun will use up knowing that the efficiency of nuclear reactions is something like 0.007:

$$eff = \frac{E_{\text{produced}}}{m\,c^2}$$

$$m = \frac{E_{\text{produced}}}{eff\ c^2}$$

$$m = \frac{3.8 \times 10^{26}\,\text{J}}{(0.007)\,(3.00 \times 10^8\,\text{m}\,\text{s}^2)}$$

$$m = 6.032 \times 10^{11}\,\text{kg}$$

   That's how much mass of fuel the Sun uses in a second, so the rate at which it is using up fuel is $6.032 \times 10^{11}\,\text{kg}\,\text{s}^{-1}$. (Note: I'm keping more sig figs than I really have here, since I will use this number in subsequent calculations.) (Note 2: this is *not* the rate at which the Sun is converting mass to energy. Remember that when you fuse Hydrogen to make Helium, only about 0.7% of the mass is lost to energy. Most of the mass stays around, only now it's in the form of Helium, and is no longer fuel for the Sun in it's current state.)

   Now we can figure out the lifetime of the Sun:

$$t = \frac{1.99 \times 10^{30}\,\text{kg}}{6.032 \times 10^{11}\,\text{kg}\,\text{s}^{-1}}$$

$$t = 3.299 \times 10^{18}\,\text{s}\left(\frac{1\,\text{yr}}{3.156 \times 10^7\,\text{s}}\right) = 1.0 \times 10^{11}\,\text{yr}$$

   This gives us a lifetime of 10 billion years. This is curious to me, becuase that is the *real* lifetime of the Sun, yet the Sun will *not* really use all of its mass as fuel. However, we also haven't taken into account the fact that over it's lifetime, the Sun does get a bit brighter with time, so perhaps that makes enough difference.

2. This problem is a little different. It's tempting to go back and repeat the calculation of the previous problem for Vega, but that is both more work than is needed, and potentially wrong! The only reason it would work is that the 10 billion figure I give you here is the same as the answer you go t in the last problem. Let's assume, howver,

that the answer to the previous problem was something different. We still have to assume that Vega will use the same *fraction* of its available fuel as the Sun will, and that Vega's nuclear reactions are equally efficient as the Sun's. The latter assumption should be good, since Vega is doing the same kind of fusion as the Sun right now.

We don't have a rate of fuel use for either star, but we do know that the rate of fuel use is proportional to the luminosity of each star. Indeed, we know from the reasoning of the previous problem that:

$$\text{rate of fuel use} \;=\; \frac{L}{eff\;c^2}$$

c is just a constant, and if eff is the same for the Sun and Vega, then we know:

$$\frac{\text{Sun's rate of fuel use}}{\text{Vega's rate of fuel use}} \;=\; \frac{L_\odot}{L_V}$$

Take the two rate equations and divide them by each other; cancelling out the stuff that cancels out (c and eff), you end up with:

$$\frac{t_V}{t_\odot} \;=\; \left(\frac{M_V}{M_\odot}\right)\left(\frac{L_\odot}{L_V}\right)$$

The ratio of masses is proportional the ratio of amounts of fuel (if the Sun will use the same fraction of its mass as fuel as wlil Vega). The ratio of lumonisites is proportional to the rates of fuel use.

We can put in the ratios we know:

$$\frac{t_V}{t_\odot} \;=\; (3)\left(\frac{1}{100}\right)$$

$$t_V \;=\; 0.03\,t_\odot$$

$$t_V \;=\; 3\times 10^8\,\text{yrs}$$

Insofar as our assumptions are correct, we expect Vega to only last about 300 million years, which is a much shorter lifetime than the Sun will have.

3. This is exactly the same problem as the previous problem, only the numbers are different. Through the same reasoning, we get:

$$\frac{t_{\text{BS}}}{t_\odot} \;=\; (0.2)\,(100)$$

$$t_{\text{BS}} \;=\; 20\,t_\odot$$

$$t_{\text{BS}} = 2.0 \times 10^{12}\,\text{yrs}$$

We expect Barnard's Star to have a lifetime of 2 trillion years, which is a lot longer than the lifetime of the Sun.

4. Nope! The Universe is only 13.7 billion years old, so no matter how soon after the beginning of the Universe we made a Barnard's Star-like star, not enough time has elapsed to have gone through that whole lifetim!

5. Sure! The Sun only lasts 10 billion years, and the Universe is 13.7 billion years old. If any Sun-like stars were made during the first 3 or so billion years, they would have time to go through their entire lifetime by now. (And there were stars made that early.)

6. Yeah, yeah, I know, in the solutions to group problems #1 I mucked up some of the numbers. I'll do it right here:

   (a)

$$\left(\frac{1 \times 10^{36}\,\text{protons}}{\text{s}}\right)\left(\frac{1.7 \times 10^{-27}\,\text{kg}}{\text{proton}}\right)\left(\frac{3.156 \times 10^{7}\,\text{s}}{\text{year}}\right)\left(1 \times 10^{6}\,\text{year}\right)$$
$$= 5.4 \times 10^{22}\,\text{kg}$$

   Notice that all the units cancel except for kg. This is a rate problem combined with unit conversion. I start with the rate in protons/second, convert it to a mass rate given the mass of the proton, convert it from "per second" to "per year", and then multiply the rate by the total time (a million years) to get the total mass lost over a million years.

   (b)

$$f = \frac{5.4 \times 10^{22}\,\text{kg}}{1.99 \times 10^{30}\,\text{kg}}$$

$$f = 2.7 \times 10^{-8}$$

   Notice there are no units on this number; it's just a ratio, just a fraction. Over a million years, the Sun loses about 3 hundred millionths of is mass to the Solar Wind. (I.e. not very much, although it's a lot in absolute terms compared to everyday human experience.)

   (c) The Sun is producing 3.8×10^{26} Joules of energy every second. The amount of energy produced is equal to $E = mc^2$, where m is the amount of mass converted to energy. So the amount of mass lost every second is:

$$m = \frac{E}{c^2} = \frac{3.8 \times 10^{26}\,\text{J}}{3.00 \times 10^{8}\,\text{m}\,\text{s}^{-1}} = 4.222 \times 10^{9}\,\text{kg}$$

   Thus, the rate at which mass is being converted to energy in the Sun is $4.2 \times 10^{9}\,\text{kg}\,\text{s}^{-1}$. (Can you explain why this rate is different from the $6.0 \times 10^{1}1\,\text{kg}\,\text{s}^{-1}$ rate we got for the Sun in Problem 1?)

In a million years, the amount of mass converted to energy through fusion is:

$$\left(4.222 \times 10^9 \,\frac{\text{kg}}{\text{s}}\right) \left(1 \times 10^6 \,\text{yr}\right) \left(\frac{3.156 \times 10^7 \,\text{s}}{1 \,\text{yr}}\right)$$
$$= \; 1.333 \times 10^{23} \,\text{kg}$$

We want the fraction this is of the Sun's mass:

$$f \;=\; \frac{1.333 \times 10^{23} \,\text{kg}}{1.99 \times 10^{30} \,\text{kg}}$$

$$f \;=\; 6.7 \times 10^{-8}$$

The Sun is actually losing *more* mass by converting mass to energy through $E = mc^2$ than it is by shedding particles in the Solar Wind!