Astro 102 Group Problems #3
Solutions

1. **(a)** The minimum angle difference measurable is $0.05''$. The actual angular displacement you measure for a star is twice the parallax angle p, so the minimum parallax angle p you can measure is $0.025''$. This corresponds to a parallax of:

$$d = \frac{1}{p} = \frac{1}{0.025} = \boxed{40\,\mathrm{pc}}$$

(b) This is an *upper* limit. You can only measure parallaxes this size or larger. A bigger parallax angle is a closer object (d goes down as p goes up).

2. Well, first, we can easily get the distance to Betelgeuse:

$$d = \frac{1}{0.076} = 131.58\,\mathrm{pc}$$

(keeping extra digits on a number that will be used in subsequent calculations).

Now, we have the distance, so we can use the small angle formula to figure out the angular size of Betelgeuse. However, that formula requires an angular size in radians, and we have it in arcseconds. So, let's convert:

$$A = 0.06'' \left(\frac{1\,\mathrm{rad}}{206,265''}\right) = 2.91 \times 10^{-7}\,\mathrm{rad}$$

Now let's use the small angle formula:

$$A = \frac{h}{d}$$
$$h = A\,d = 2.91 \times 10^{-7}\,131.58\,\mathrm{pc} = 3.83 \times 10^{-5}\,\mathrm{pc}$$

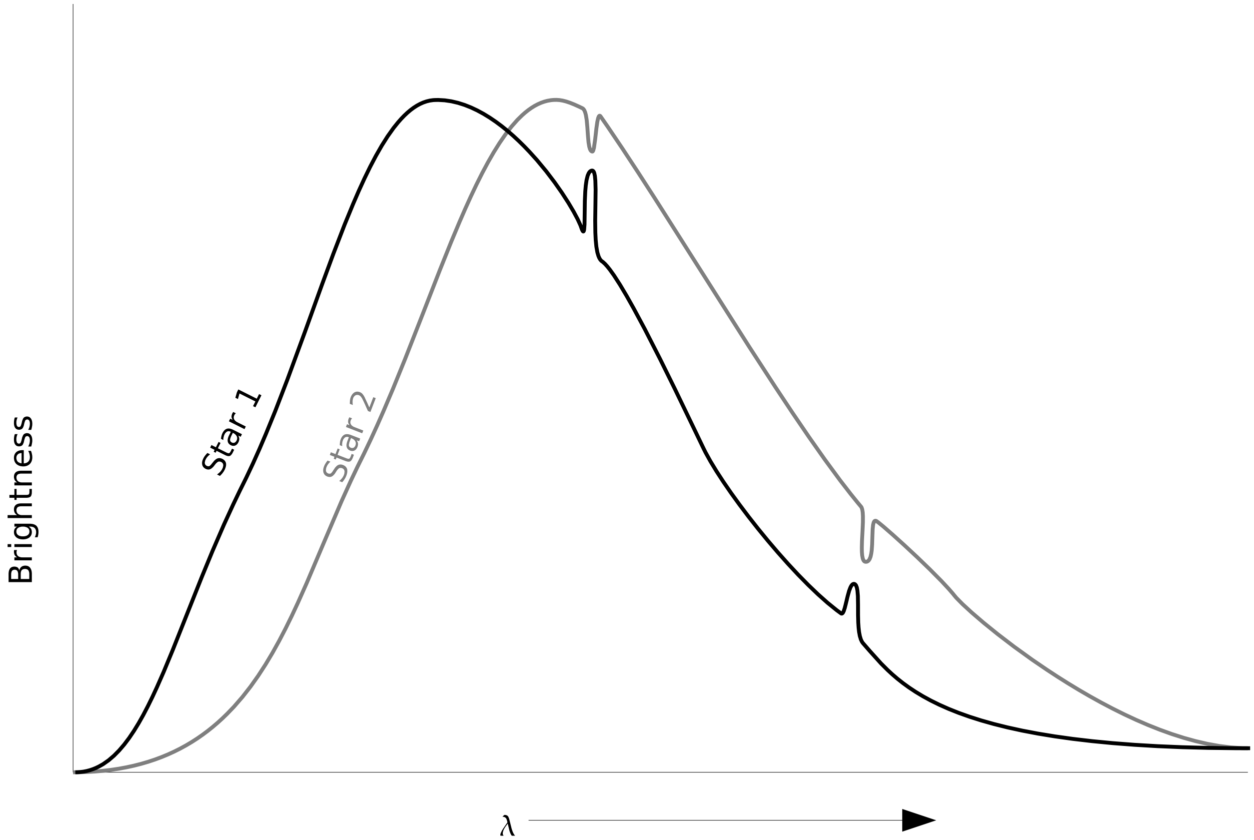
That's the diameter of Betelgeuse. Let's convert this to AU and then km for comparison with other things:

$$h = 3.83 \times 10^{-5}\,\mathrm{pc} \left(\frac{206,265\,\mathrm{AU}}{1\,\mathrm{pc}}\right) 7.9\,\mathrm{AU}$$

$$h = 7.89\,\mathrm{AU} \left(\frac{1.496 \times 10^{8}\,\mathrm{km}}{1\,\mathrm{AU}}\right)$$
$$h = 1.2 \times 10^{9}\,\mathrm{km}$$

Dividing this by the diameter of the Sun (which is twice the Sun's radius), we see that this is about 850 times the radius of the Sun (it's a monster). The radius of Betelgeuse is 3.9 AU, which is 3.9 (i.e. about 4) times the radius of Earth's orbit around the Sun.

3.



4. (a) It's that L-B-d equation again!

$$
\begin{aligned}
B &= \frac{L}{4\pi\, d^2} \\
L &= B\, 4\pi\, d^2 \\
\\
L_2 &= 5000\, L_\odot \\
\\
B_2\, 4\pi\, {d_2}^2 &= 5,000\, B_\odot\, 4\pi\, {d_\odot}^2
\end{aligned}
$$

Solve this for d_2:

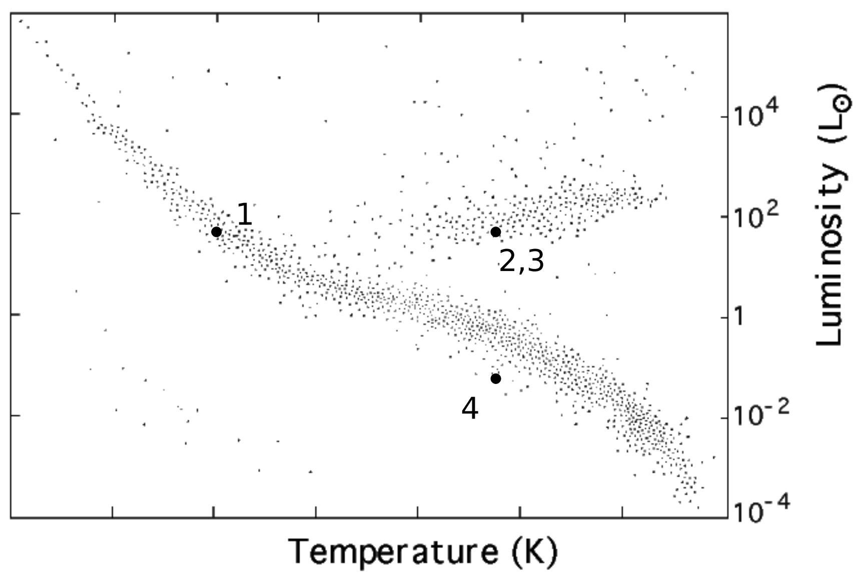
$$
\begin{aligned}
d_2 &= d_\odot \sqrt{\frac{5,000\ B_\odot}{B_2}} \\
d_2 &= 1\,\mathrm{AU}\sqrt{5000 \times 8.5 \times 10^9} \\
d_2 &= 6.519 \times 10^6\,\mathrm{AU}\left(\frac{1\,\mathrm{pc}}{206,265\,\mathrm{AU}}\right) = \boxed{32\,\mathrm{pc}}
\end{aligned}
$$

(b) Now it's the L-R-T equation!

$$
\begin{aligned}
L_2 &= 5,000\, L_\odot \\
4\pi\, {R_2}^2\, \sigma\, {T_2}^4 &= 5,000\,(4\pi)\, {R_\odot}^2\, \sigma\, {T_\odot}^4 \\
R_2 &= \sqrt{5000}\, R_\odot \left(\frac{T_\odot}{T_2}\right)^2 \\
\end{aligned}
$$

$$\boxed{R_2 = 34\,\mathrm{R}_\odot}$$

5.



6. Jupter is 5.2 times farther from the Sun than the Earth, so from Jupiter, the Sun is 5.2^2 times, or 27 times, less bright. Thus, the ratio of the brighness of the Sun to Star 2 is:

$$\frac{8.5 \times 10^9}{27} = \boxed{3.2 \times 10^8}$$