Astro 102
Group Problems #4 Solutions
2006-June-29

1.

$$
\begin{aligned}
d &= z\, c\, t_H \\
d &= 0.021 \left(1\, \frac{\mathrm{lyr}}{\mathrm{yr}}\right) \left(13.8 \times 10^9\, \mathrm{yr}\right)
\end{aligned}
$$

$$
\boxed{d = 2.9 \times 10^8\, \mathrm{lyr} = 290\, \text{million light years} = 89\, \mathrm{Mpc}}
$$

2. If we define t_{LB} as the lookback time, we need $t_{\mathrm{LB}} = 250 \times 10^6\, \mathrm{yr} = 0.25$ billion years.

$$
z = \frac{\frac{d}{c}}{t_H} = \frac{t_{\mathrm{LB}}}{t_H}
$$

since d is the distance the light has to travel, and c is the speed the light is going.

$$
z = \frac{0.25\, \mathrm{Gyr}}{13.8\, \mathrm{Gyr}}
$$

I use the t_{LB} in billions of years so that the units will cancel. Recall that z is just a number, and shouldn't have units.

$$
\boxed{z = 0.018}
$$

Nuts! VV114 is just a bit too far away.

3. **(a)** Recall that $1+z$ is the current size divided by the size when the light was emitted. We need $1+z=3$, so $\boxed{z=2}$.

(b) We have

$$
\begin{aligned}
1 + z &= \frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{emit}}} \\
\lambda_{\mathrm{obs}} &= (1+z)\, \lambda_{\mathrm{emit}} \\
\lambda_{\mathrm{obs}} &= (3)\, (4861\, \text{Å})
\end{aligned}
$$

$$
\boxed{14580\, \text{Å} = 1.458\, \mu\mathrm{m}}
$$

(c) This is in the infrared region of the spectrum. (In fact, it's in the near-infrared. This is the region I was applying for time to look at VV114 with the Gemini telescope, but I got turned down. Am I bitter? Noooo....)

(d) The neutron star to white dwarf ratio will be *higher*. We're looking at a galaxy when it is very young — the lookback time is a lot, approaching the age of the Universe. (In fact, the lookback time is 11.5 billion years, but I wouldn't expect you to be able to figure that out.) There *will be* some white dwarfs in this galaxy, since the most massive star that makes a white dwarf ($8\,\mathrm{M}_\odot$) only lives a few times 10 million years, so there has been time to start making white dwarfs. However, in our galaxy, we've been able to turn stars that live as long as 10 billion years into white dwarfs, whereas this distant galaxy will only be able to turn stars that live up to 2 billion years into white dwarfs. Shorter lived and less massive stars rae more common, so there is a greater pool of possible "white dwarf progenitors" in our Galaxy than there is in this galaxy seen earlier in the Universe's history.

Note that our Galaxy will have, in an absolute sense, *more* neutron stars, since it's been making high mass stars all along that have turned into neutron stars. However, the difference is *greater* for white dwarfs, because there has been more time to allow the more common, lower mass stars to go through their lives.

4. We want the redshift of the second galaxy. We could use the Hubble Law to find this *if* we knew the distance. We don't, however.

We do have the redshift of the first galaxy, so we can find that galaxy's distance. What's more, we have a 10-day period Cepheid variable in each galaxy; because they have the same period, the two Cepheid variables have the same luminosity. As such, we know:

$$\begin{aligned} L_{C1} &= L_{C2} \\ B_{C1}\,4\pi\,{d_{C1}}^2 &= B_{C2}\,4\pi\,{d_{C2}}^2 \end{aligned}$$

$$d_{C2} = d_{C1}\sqrt{\frac{B_{C1}}{B_{C2}}}$$

We also know that $d_{C1} = z_{C1}\,c\,t_H$, so we have:

$$d_{C2} = z_{C1}\,c\,t_H\sqrt{\frac{B_{C1}}{B_{C2}}}$$

What we really want, though is z_{C2}, but now we have d_{C2} in terms of stuff we know, we can use the Hubble Law:

$$\begin{aligned} z_{C2} &= \frac{d_{C2}}{c\,t_H} \\ z_{C2} &= \frac{z_{C1}\,c\,t_H}{c\,t_H}\sqrt{\frac{B_{C1}}{B_{C2}}} \\ z_{C2} &= z_{C1}\sqrt{\frac{B_{C1}}{B_{C2}}} \end{aligned}$$

Notice that we don't need to know t_H at all!

$$
\begin{aligned}
z_{C2} &= 0.001\sqrt{\frac{100}{1}} \\
\boxed{z_{C2} = 0.01}
\end{aligned}
$$

5. Here's how I'm going to approach this. The Hubble Time is what characterizes the expansion rate of the Universe. So, let's first calculate the Hubble time that we would find from each Galaxy, and then compare them to each other.

For the first galaxy, we have:

$$
z = \frac{d}{c\,t_H}
$$

which gives us

$$
t_H = \frac{d}{c\,z} = \frac{550\,\mathrm{Mlyr}}{(1\,\mathrm{lyr\,yr^{-1}})\,(0.04)} = 13.75\,\mathrm{Gyr}
$$

Hmm, 13.8 billion years. . . sounds like the instructor chose his numbers wisely here, eh? Note above that I just used the distance of 550 Mlyr based on the fact that the lookback time is 550 Myr.

Right-o. Second galaxy. This one's harder, since we don't have a distance, but since a Type Ia supernova is a standard candle, we can use it to figure out the distance:

$$
\begin{aligned}
L_{SN1} &= L_{SN2} \\
B_{SN1}\,4\pi\,{d_{SN1}}^2 &= B_{SN2}\,4\pi\,{d_{SN2}}^2 \\
d_{SN2} &= d_{SN1}\sqrt{\frac{B_{SN1}}{B_{SN2}}} \\
d_{SN2} &= 550\,\mathrm{Mlyr}\,\sqrt{4} \\
d_{SN2} &= 1,100\,\mathrm{Mlyr}
\end{aligned}
$$

Now we can calculate an expansion rate (given by t_H) for the second galaxy:

$$
t_H = \frac{d}{c\,z} = \frac{1,100\,\mathrm{Mly4}}{(1\,\mathrm{lyr\,yr^{-1}})\,(0.08)} = 13.75\,\mathrm{Gyr}
$$

Hmm, same thing. No difference in the expansion rate when we look over the last 1,100 milion years and when we look over the last 550 years.

Next galaxy:

$$
\begin{aligned}
L_{SN1} &= L_{SN3} \\
B_{SN1}\,4\pi\,{d_{SN1}}^2 &= B_{SN3}\,4\pi\,{d_{SN3}}^2 \\
d_{SN3} &= d_{SN1}\sqrt{\frac{B_{SN1}}{B_{SN3}}}
\end{aligned}
$$

$$\begin{aligned} d_{SN3} &= 550\,\text{Mlyr}\sqrt{\frac{1}{0.0625}} \\ d_{SN3} &= 2,200\,\text{Mlyr} \end{aligned}$$

Now we can calculate an expansion rate (given by t_H) for the third galaxy:

$$t_H = \frac{d}{c\,z} = \frac{2,200\,\text{Mly4}}{(1\,\text{lyr}\,\text{yr}^{-1})\,(0.18)} = 12.2\,\text{Gyr}$$

Hey! This is different! So is the Universe speeding up or slowing down?

If you think about it, a *lower* t_H implies a faster expansion. Recall that t_H can be interpreted as the amount of time it would take the size of the Universe to double if the expansion rate stays constant. If that time is lower, then the expansion rate must be faster.

Looking at the amount of expansion in the last 2.2 billion years, we get a faster expansion rate than we do looking at the amount of expansion only over the last 0.5–1 billion years. This tells us that the expansion of the Universe is $\boxed{\text{slowing down}}$.

Note that the numbers in this problem are not real. The expansion of the Universe is in fact speeding up. What's more, the acceleration of the Universe is so small that it is impossible to see with the precision we can measure out to redshifts less than about 0.2. As such, the huge effect given in this problem is not only in the wrong directon, but a vast exaggeration of what is really seen.

6. Recall that if the expansion rate has been constant, then t_H is the age of the Universe. To figure this out, remember that:

$$\frac{\text{change in size}}{\text{starting size}} = \frac{t}{t_H}$$

(This is the standard "doubling time" rate equation.) If we pick some distant galxay whose distance right now is d, we need the change in distance to be $-d$. Then we will get the time (which will be negative, since we're looking in the past) that the two galaxies were arbitrarily close together — that's the Big Bang.

$$\begin{aligned} \frac{-d}{d} &= \frac{t}{t_H} \\ t &= -t_H \end{aligned}$$

So, if the expansion rate has been constant, then t_H is the age of the Universe.

However, if the expansion has been *slowing down*, then it was *faster* in the past. This means that we got to where we are now in *less* time than we would have if the expansion rate had been constant. (Consider an analogy: a car is currently going 60 mph, and has gone 120 miles. If, however, it was going 90 mph for the first 60 miles, did it take the car more than, equal to, or less than 2 hours to complete the trip?)

If the Universe has only decelerated, it's always been the current rate or faster. In this case, then the actual age of the Universe would be *less than* the Hubble Time.