Astro 260
Group Problems #2
2005-January-28

Consider the geometry described by the following line element:

$$ds^2 = \left(1 - \frac{2\,\Phi(\vec{r})}{c^2}\right)(dr^2 + r^2\,d\phi^2)$$

where c is the speed of light and Φ is the Newtonian gravitational potential. Use the potential for our solar system:

$$\Phi(\vec{r}) = \frac{-G\,M_\odot}{r}$$

where $G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$ and $M_\odot = 2.00 \times 10^{30}\,\mathrm{kg}$.

(a) Write down the Lagrangian for a test particle moving in this geometry under this potential.

(b) Derive the r and ϕ equations of motion from this Lagrangian.

(c) Explicitly identify how these equations differ from the equations you get in flat space ($ds^2 = dr^2 + r^2 d\phi^2$).

(d) For the Earth (orbiting at a distance of $1\,\mathrm{AU} = 1.496 \times 10^{11}\,\mathrm{M}$), by how much do these equations differ from the flat-space equations? (I.e. are they of the same order, are the additional terms smaller by 10^{-3}, etc.?)

(e) What condition on the radius of the orbit r would need to be satisfied for these terms to start to be significant?

(f) Compare the solutions you get for something in an elliptical orbit starting with ($r = 1\,\mathrm{AU}$, $\dot{r} = 0$, $\phi = 0$, and $\dot{\phi} = 5\,\mathrm{rad/year}$) for the flat and curved spaced values. (Obviously, doing this numerically requires a computer, and I don't really expect anybody to get this far in class.)

(g) Compare the solutions you get for something in an elliptical orbit starting with ($r = 10^{-6}\,\mathrm{AU}$, $\dot{r} = 0$, $\phi = 0$, and $\dot{\phi} = 5 \times 10^9\,\mathrm{rad/year}$) for the flat and curved spaced equations. Qualitatively how could you describe the orbit in the curved space in comparison to the ellipse you get in flat space?

This geometry is the *spatial* component of the line element you get in the Newtonian limit (the not-within-your-answer-to-part-e limit) of GR. However, it's not entirely physcially reasonable, because we've left out a term on the time component of the spacetime line element, and have assumed standard absolute time.