

Astro 260
Group Problems #6
2005-February-25

1. Consider the static weak-field metric for a spherically symmetric gravitational field arising from a body of mass $M_{\oplus} = 6.0 \times 10^{24}$ kg (i.e., the Earth). Suppose that somebody is at constant radius $r = R$ but is orbiting with constant angular velocity ω as measured by a very distant observer. (For simplicity, at some point you will probably want to set $\theta = \pi/2$. The motion will be in a plane, and since we have spherical symmetry, you have the freedom to choose θ wherever it is.)

Just in case you care, $G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ and $c = 3.00 \times 10^8 \text{ m s}^{-1}$.

- (a) If you want to go into the frame of reference of the somebody, using coordinates (t', r', θ', ϕ') , what would be a simple coordinate transformation you could perform? (HINT: this is easy; the coordinate transformation is very straightforward and what you would expect even without relativity. The coordinates you are looking for *aren't* necessarily the “proper” coordinates of the somebody, but things that will make calculations possible. Consider that we used the static weak-field metric to do calculations of time dilation on the surface and in orbit of Earth, even though the coordinates in that metric weren't proper time/length at either of those two locations.)
- (b) Write down the line element for the somebody in terms of t', r', θ', ϕ' .
- (c) To stay in the static weak-field limit, not only do we need $2GM/r \ll c^2$, but we also need $wr \ll c$. Given this, convert your answer to b so that it is only to “first order in $1/c^2$, i.e. so that it doesn't have any (dinky)² terms in it. (You may notice that there is a (dinky)^{1/2} term in your expansion...but still keep the first-order (dinky) terms.)
- (d) For what observer are the coordinates $(t', r', \theta', \text{and } \phi')$ coordinates of proper time and proper distance?
- (e) What is the time dilation factor $d\tau/dt'$ if the somebody is standing on the equator of the Earth ($R_{\oplus} = 6,400$ km)? (You should be able to figure out ω from numbers you just know....) Write your answer in the form $1 \pm \epsilon$, because your answer will be 1 to any reasonable number of significant figures. Which effect dominates, gravity or special relativity?
- (f) What is the time dilation factor $d\tau/dt'$ if the somebody is orbiting at an altitude of 400km? (Use standard classical Newtonian gravity to figure out ω in this case.) Which effect dominates, gravity or special relativity?
- (g) What is the ratio $d\tau(\text{orbit})/d\tau(\text{surf})$ between the time dilation for the two observers of the previous two parts? Is it > 1 or < 1 ? Is this consistent with our previous conclusion that the orbiting clocks should be running slower?