A260 Homework #1
Due Monday, 2005-Jan-31
(at the beginning of class)

1. *[Hartle problem 3.2]*

2. Consider a two-dimensional spherical geometry described by the line element:

$$ds^2 \;=\; a^2\, d\theta^2 \;+\; a^2 \sin^2\theta\, d\phi^2$$

where a is a constant. Consider a particle of mass m moving in this two-dimenaional space. (If you don't like two-dimensional spaces, remember that even in Euclidean geometry, the motion of particles under many force fields is confined to a plane, e.g. the orbit of a planet around the Sun.) This particle feels a potential (from some unidentified force; perhaps the electric force) that takes the form:

$$V \;=\; a\, k\, \theta$$

where k is some constant.

(a) Write down the Lagrangian for this system.

(b) From the Lagrangian, determine and write down the equations of motion (differential equations in θ and ϕ as a function of t, potentially coupled) for this system.

(c) Where on the sphere (what values of θ and/or ϕ) is this potential most like the potential that we see in a uniform force field, e.g. the gravitational field for person-sized objects near the surface of the Earth? Show that the equations of motion reduce to the familiar equations of motion for small displacements from this position on the sphere.

(d) [Long] Numerically solve the equations from part (b) (using Mathematica, your own code, or your favorite way for numerically solving DEs). Make three plots: $\theta(t)$, $\phi(t)$, and $\theta(\phi)$. On each plot, draw two paths: one for a particle in a flat space moving with potential $V = -ky$ (with $y = a\theta$ and $x = a\phi$), and another for the equations of motion from (b), at the position on the sphere you identfied in (c). Choose equivalent initial conditions. (The point of this is to see how much the particle path deviates as a result of being in a curved space.)

3. ***(Solo Problem)*** You are falling directly towards the Sun (not orbiting it), in a closed laboratory. Your laboratory falls freely under the influence of gravity. You start at rest 1 AU distant from the Sun, and begin to fall from there. However, in your closed laboratory, you know none of this; you just think you're in an inertial frame. Consider only Newtonian gravity.

You start with two balls in the air, 1m apart. They are separated along the radial direction (with the origin on the Sun).

(a) What is the tidal acceleration of the separation of the two balls, in m/s^2, when your fall starts?

(b) What is the tidal acceleration on the separation of the two balls (assume you have replaced them to a 1m separation, if necessary) moments before you hit the surface of the Sun and burn up?

(c) What is the acceleration of your laboratory moments before it burns up?

Some things you may want to know:

- $1\,\mathrm{AU} \;=\; 1.49 \times 10^{11}\,\mathrm{m}$ (Sun-Earth separation)
- $M_\odot \;=\; 2.00 \times 10^{30}\,\mathrm{kg}$ (mass of the Sun)
- $R_\odot \;=\; 6.96 \times 10^{8}\,\mathrm{m}$ (radius of the Sun)

Hint: if you try to solve for each ball's path separately and subtract the answers, you don't have enough numerical precision to figure out what's going on. You're better off thinking about the difference in acceleration of the two balls (derivative of gravity as a function of distance from the Sun), and getting an expression for Δa, the relative acceleration of the two balls.