A260 Homework #3

Due Monday, 2005-Feb-28 (at the beginning of class)

1. ***(Solo Problem)*** Consider three reference frames. Call the unprimed frame the "lab" frame. The primed frame is moving at speed v in the $+x$ direction with respect to the lab frame. The double-primed frame is rotated by angle θ around the lab frame's z-axis, but is at rest with respect to the lab frame.

   (a) One can envision a transformation $R^{\alpha''}{}_{\beta}$ to convert coordinates in the lab frame to the double-primed (rotated) frame. Write down the sixteen coefficients $R^{\alpha''}{}_{\beta}$ of this transformation. (You can also think of $R^{\alpha''}{}_{\beta}$ as a transformation matrix. If you write it in matrix form, make sure you're very clear as to which index is counting rows, and which is counting columns.)

   (b) Recall that the Lorentz Transformation between the lab and primed frame is:

$$\Lambda^{\alpha'}{}_{\beta} = \begin{bmatrix} \gamma & -\gamma v & 0 & 0 \\ -\gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (where, thanks to the symmetry, I don't have to tell you which index counts over rows and which counts over columns; the transpose would be the same matrix!). Also recall that the inverse transformation is:

$$\Lambda^{\alpha}{}_{\beta'} = \begin{bmatrix} \gamma & \gamma v & 0 & 0 \\ \gamma v & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   You don't have to write anything for this part, just recall all that.

   (c) $\Lambda^{\alpha''}{}_{\beta'}$ is the transformation from the primed (rotated but at rest with respect to the lab) frame to the double-primed (moving) frame. Write down an expression in terms of the $R^{\alpha''}{}_{\beta}$ and either $\Lambda^{\alpha}{}_{\beta'}$ or $\Lambda^{\alpha'}{}_{\beta}$. (Don't expand out the coefficients of these.) (Hint: you can change Greek letters where you need to, so long as the rules of the summation convention are obeyed. Indeed, if the rules require you to change letters, you really should.)

   (d) Write down all the elements of $\Lambda^{\alpha''}{}_{\beta'}$. If you write it in matrix form, be sure to make clear which index counts over rows, and which counts over columns.

2. ***(Solo Problem)*** Consider the three reference frames of the previous problem. Consider a rhinoceros moving at speed u in the $+y$ direction as measured in the *primed* frame.

   (a) What are the components $u^{\alpha'}$ of the rhino's 4-velocity in the primed frame?

   (b) What are the components $u^{\alpha''}$ of the rhino's 4-velocity in the double-primed frame?

   (c) Verify that the normalization condition on $\mathbf{u}$ is satisfied for your answer to (b).

...continued...

3. Consider the static weak-field metric (eq. 6.20 in Hartle). Consider the $\Phi(x^i)$ for the Earth. Ignore the Sun, the Galaxy, the orbit of the Earth, the rotation of the Earth, the rotation of the Galaxy, Harry Boxlee's playtime, etc. for purposes of this problem.

   (a) Consider two points, one on the surface of the Earth, one a distance 300km (or 0.001 light-seconds) directly above it. Call Δr the difference in the r coordinate value of these two points. What is the value of $(\Delta r - 300\text{km})$?

   (b) By what factor $d\tau/dt$ are you time dilated with respect to a very distant observer (who's $d\tau$ is equal to dt)? (Express your answer as $1-$ something, since to any reasonable number of significant figures the answer itself is just gonna be 1, which isn't interesting.)

   (c) OK, I lied. Consider the Sun, and consider if you were at the same distance from the Sun the Earth is, but nowhere near Earth (so you don't have to worry about Earth's gravity for this part of the problem). (Continue to assume that everything is at rest and not orbiting about anything, which is of course nutty, but hey, the purpose of all of this problem is to play around with gravity, not special relativity.) By what factor $d\tau/dt$ are you time dilated with respect to a very distant observer?

   Look up numbers as needed. You may or may not be surprised to find out they are all in the Particle Data Book.... Yes, the Sun is now a particle.

4. Hartle problem 6.6.

5. Hartle problem 6.13.