A260 Homework #5
Due Monday, 2005-Apr-25
(at the beginning of class)

1. *Hartle 18.6.* (This one should be easy.)

2. Derive an expression for *special relativistic* time dilation $dt/d\tau$ (where dt is the passage of time measured by an observer, and $d\tau$ is the passage of proper time for the object under observation), in terms of z (the redshift of the object under observation as measured by the observer).

3. *(Originally part of Group Problems #9.)* When cosmologists actually make observations, they define other "distances" which aren't distances at all, but things with units of distance that are conveniently matched to things they can observe. The *luminosity distance* is defined such that:

$$F = \frac{L}{4\pi\, {d_L}^2}$$

where L is the instrinsic luminosity (J/s) of an object, and F is the measured flux from that object (J/(s m^2)). F is, of course, directly measurable; it's what comes out as brightness when you point a telescope at something. You should recognize that in a flat, Euclidean, non-expanding, static space, d_L is exactly the same as the physical distance to an object.

For this problem, you may work entirely in the flat Robertson-Walker metric, although it isn't too difficult to extend the consideration to the general Robertson-Walker metric. Do the latter if you feel so inspired.

(a) If L is the luminosity of the emitting object, what is dN/dt', the number of photons emitted by that object in its own proper time dt'? (For simplicitly, assume all the photons are of the same frequency.)

(b) If the object is at *coordinate* distance r, what is A, the area of the sphere over which the photons emitted at the same time as the ones we detect from the object are spread?

(c) Write down an expression for $\frac{dN}{dA\,dt}$, where dN is the number of photons emitted by a distance source in its proper time dt' and dt is the amount of time it takes *us* to detect those photons. Your expression should be in terms of L, ω' (the frequency of the emitted photons), and r.

(d) Write down an expression for $\frac{dE}{dA\,dt}$, the flux we detect, in terms of the quantities from (c) and ω, the detected photon frequency.

(e) Convert your expression in (d) to an expression in terms of z, Ω_{M}, Ω_Λ, and H_0. This is the standard *luminosity distance* formula. By measuring F and z for a lot of objects of known L, we can fit the values of the parameters H_0, Ω_{M}, and Ω_Λ to this curve.

. . . continued on reverse. . .

4. [***Solo Problem***] Start with the general Friedmann equation (18.63).

(a) What is the critical density at any time such that if the Universe has this density, it is flat ($k = 0$)? Express your answer in terms of a, $\dot{a}$, and constants. Compare this to the *present* value of the critical density (18.32).

(b) Show that if the Universe has the critical density at one point, then it always has the critical density (i.e. a flat Universe stays flat). That is, show that:

$$\frac{d}{dt}\left(\frac{\rho}{\rho_C}\right) = 0$$

where ρ_C is the critical density as a function of time (i.e. your answer to a). (Do this by solving for the derivative of ρ/ρ_C in a general Universe, and then by arguing that the derivtive is zero in the special case of a flat Universe.)

(c) Show that if the Universe is *expanding*, and the expansion is *decelerating*, then Ω_{tot} diverges from 1; that is, the density of the Universe gets farther from the critical density.