Astro 260 Group Problems
2007-February-27

1. *A Theory of Everything*

   (a) Develop a working theory of quantum gravity.

   (b) Show that your theory reduces to Newton's Laws and Maxwell's Equations in the appropriate limit.

2. Consider two observers outside a black hole. $\mathcal{O}_1$ is at Schwarzschild coordinate $r = R_1$, and $\mathcal{O}_2$ is at Schwarzschild coordinate $r \gg 2M$ (assume $R_2 = \infty$).

   We showed in class that the gravitational redshift for a photon emitted by $\mathcal{O}_1$ radially to $\mathcal{O}_2$ is

   $$\frac{\omega_2}{\omega_1} = \left(1 - \frac{2M}{R_1}\right)^{\frac{1}{2}}$$

   using the Schwarzschild metric. The same geometry is described also by the Eddington-Finkelstein metric:

   $$ds^2 = -\left(1 - \frac{2M}{r}\right) dv^2 + 2dvdr + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2$$

   Show explicitly that if you do the gravitational redshift calculation above using this metric, you get the same answer. (If you don't, it means that either you made a mistake, or this metric doesn't really describe the same spacetime geometry as the Schwarzschild metric.)

3. We have seen and used the fact that if an observer moving with 4-velocity $\mathbf{u}$ observes a object (either massive like a particle or massless like a photon) with 4-momentum $\mathbf{p}$, then the energy that observer measures is $E = -\mathbf{u} \cdot \mathbf{p}$.

   (a) Show that the *magnitude* of the 3-momentum (what we usually think of as just "momentum" in special relativity) measured by the observer has the value:

   $$|\vec{p}| = \left[(\mathbf{p} \cdot \mathbf{u})^2 + \mathbf{p} \cdot \mathbf{p}\right]^{\frac{1}{2}}$$

   (b) Define (for purposes of this problem only) a four-vector $\mathbf{v}$ that has components $v^0 = 0$ and $v^j = dx^j/dt$ in the observer's local Lorentz frame. That is, the time component is 0, and the three space components are the normal 3-velocity. Show that this vector is given by:

   $$\mathbf{v} = \frac{\mathbf{p} + (\mathbf{p} \cdot \mathbf{u})\mathbf{u}}{-\mathbf{p} \cdot \mathbf{u}}$$